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LINE OF SIGHT CONSTANTS FOR THE PRINCIPAL STARS.

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THE following formulae and tables are intended to facilitate the elimination of the Earth's orbital velocity from observations for motion in the line of sight. Let

λ, β, be the mean longitude and latitude of the observed star.

$\odot$, the Sun's mean longitude at the moment of observation.

Γ, the Sun's longitude at perigee (281° 20′).

a, the semi-axis major of the Earth's orbit (149,480,000 kilometers).

e, the eccentricity of the Earth's orbit (0.0167).

$90° - i$, the angle between the Earth's direction of motion and the radius vector of the orbit.

T, the Earth's sidereal period (31,558,000 mean solar seconds).

v, the *mean* velocity of the Earth in its orbit, defined more precisely below.

v_1, the Earth's velocity at the moment of observation.

x, the desired correction to the observed velocity of a star, or the projection of v_1 upon the line of sight.

From the theory of elliptical motion[1] we have

$$\tan i = \frac{e \sin (\odot - \Gamma)}{1 + e \cos (\odot - \Gamma)}$$

$$v_1 = v\,[1 + e \cos (\odot - \Gamma)] \sec i$$

[1] CHAUVENET'S *Astronomy*, 1, p. 636.

in which $$v = \frac{a}{\sqrt{1-e^2}} \cdot \frac{2\pi}{T}.$$

Projecting v_1 upon the line of sight,

$$x = -v\,[1 + e\cos(\odot - \Gamma)]\sin(\lambda - \odot + i)\cos\beta\sec i.$$

Eliminating i and simplifying we obtain

$$x = v\cos\beta\sin(\odot - \lambda) + v\,e\cos\beta\sin(\Gamma - \lambda).$$

Or, if we put

$$b = v\cos\beta$$
$$c = v\,e\cos\beta\sin(\Gamma - \lambda)$$

we have finally

$$x = b\sin(\odot - \lambda) + c.$$

The quantities b and c do not involve the longitude of the Sun and are, therefore, practically constant for any particular star. Neglecting proper motion, β cannot change more than $49''$ in a century while Γ–λ is increased about $20'$ in the same period. The maximum value of b is about 30 kilometers per second, and c never exceeds 0.50 kilometers per second. It follows that if we tabulate the values of b and c for the present epoch they will hold during the coming century without ever introducing so large an error in x as 0.01 kilometers per second.

It is interesting to note that the first term in the above expression for x is what would have resulted from the assumption of a circular orbit for the Earth. That is, although the Earth's velocity varies in both amount and direction from that of a circular orbit, yet the combined effect is constant for any particular star. The analogue to this proposition is found in the theory of aberration.[2]

In the accompanying table the constants *log* b and c have been computed for all the brighter stars visible from northern observatories. Some variables and a few stars with large proper motions are also given. The unit of velocity is the kilometer per second, and the tables and formulae conform to the usual convention of denoting approach by the negative, and recession by the positive sign. The stars are arranged in the order of increasing right ascensions. To enable a star to be readily

[2] BESSEL, *Tabulae Regiomontanae*, XIX.

found the approximate right ascension is given at frequent intervals. The longitude for 1900 and the latitude have also been tabulated, though only the former is needed in the reductions. The longitude must be brought up to the year of observation, $1900 + t$, by adding to the tabular longitude,

$$d\lambda = 0.838' \times t.$$

A short table giving the values of $d\lambda$ for every year up to 1930 will be found at the end of the paper. For a few stars with very large proper motions, such as 1830 *Groombridge*, 61 Cygni, o^2 Eridani, μ Cassiopeiae, etc., the above expression for $d\lambda$ will not suffice when t becomes large.

The value adopted for a, the semi-axis major of the Earth's orbit, is based upon 8.80″ as the solar parallax; an error of 0.01″ in this constant corresponds to a maximum error of 0.034 kilometers per second in the correction to an observed velocity. Had a different parallax been employed the only effect upon these tables would have been the addition of a constant to each *log b*. For Γ, the Sun's longitude at perigee, the round value 281° 20′ has been employed; this is about its value for 1950, referred however to the equinox of 1900.

Much assistance in checking the computations was obtained from a table of the longitudes and latitudes of six hundred stars, published by C. Chabrol in the *Connaissance des Temps* for the year XII (1804), and referred to the equinox of 1800. This table contains nearly all the stars in the present paper, and very little labor was required to bring the data up to 1900 with sufficient accuracy. However, these results were used merely as a check, all the computations being based directly upon the right ascensions and declinations of the stars referred to the equinox and epoch of 1900. The following formulae were employed:

$$\tan\lambda = [9.96255]\tan\alpha + [9.59987]\sec\alpha\tan\delta,$$
$$b = [1.47371]\sec\lambda\cos\alpha\cos\delta,$$
$$c = [8.224]\, b\sin(281^\circ\ 20' - \lambda).$$

For c a graphical check was constructed, with *log b* and λ as the arguments.

Name	Mag.	λ 1900	β	log b	c
$0^h\ 3^m$					
α Andromedae ...	2.0	12°55.0′	+25°41′	1.4285	−0.45
β Cassiopeiae	2.1	33 43.2	+51 14	1.2705	−.29
γ Pegasi	2.6	7 45.7	+12 36	1.4631	−.49
ι Ceti	3.3	359 31.1	−10 01	1.4670	−.48
T Ceti.....	Var.	335 15.0	−20 30	1.4453	−.37
$0^h\ 20^m$					
ζ Cassiopeiae	4.0	33 40.7	+44 43	1.3254	−.33
π Andromedae ...	4.0	21 17.3	+27 08	1.4230	−.44
δ Andromedae ...	3.3	20 25.2	+24 21	1.4333	−.45
α Cassiopeiae	Var.	36 23.8	+46 37	1.3106	−.31
β Ceti.....	2.0	1 10.5	−20 47	1.4445	−.46
$0^h\ 40^m$					
η Cassiopeiae	3.8	38 50.1	+47 02	1.3072	−.30
γ Cassiopeiae	2.0	42 32.6	+48 48	1.2924	−.28
μ Andromedae ...	4.0	27 46.8	+29 39	1.4128	−.42
ε Piscium	4.0	16 07.8	+1 05	1.4736	−.50
μ Cassiopeiae	5.6	39 41.5	+43 13	1.3363	−.32
$1^h\ 3^m$					
η Ceti.....	3.1	10 21.8	−16 06	1.4563	−.48
β Andromedae ...	2.3	29 00.6	+25 57	1.4276	−.43
τ Piscium	4.0	26 55.4	+20 44	1.4446	−.45
θ Ceti.....	3.0	14 49.8	−15 46	1.4570	−.48
δ Cassiopeiae	2.8	46 32.0	+46 24	1.3123	−.28
$1^h\ 20^m$					
α Ursae min.....	2.0	87 10.1	+66 06	1.0815	−.05
η Piscium	3.6	25 25.2	+5 22	1.4718	−.48
υ Persei	3.6	41 03.0	+35 24	1.3849	−.35
φ Persei	4.0	43 12.2	+36 50	1.3770	−.35
τ Ceti.....	3.3	16 27.2	−24 52	1.4315	−.45
$1^h\ 40^m$					
ζ Ceti.....	3.0	20 32.9	−20 20	1.4457	−.46
ε Cassiopeiae	3.3	53 22.4	+47 32	1.3031	−.25
α Trianguli	3.6	35 28.2	+16 48	1.4548	−.44
γ Arietis	4.0	31 47.2	+7 10	1.4703	−.46
ξ Piscium	4.0	26 07.3	−7 56	1.4695	−.47
$1^h\ 49^m$					
β Arietis	2.8	32 34.4	+8 29	1.4689	−.46
50 Cassiopeiae	4.0	62 10.3	+54 23	1.2390	−.18
υ Ceti.....	4.0	18 01.2	−31 02	1.4066	−.42
γ Andromedae ...	2.4	42 50.0	+27 48	1.4205	−.38
α Arietis	2.0	36 15.8	+9 58	1.4671	−.45
$2^h\ 2^m$					
β Trianguli	3.0	40 57.4	+20 35	1.4451	−.41
ο Ceti.....	Var.	30 07.5	−15 56	1.4567	−.45
ξ^2 Ceti.....	4.0	36 04.3	−5 52	1.4714	−.45
123 Piazzi II.....	6.6	37 23.2	−8 00	1.4695	−.44
δ Ceti.....	4.0	36 10.3	−14 28	1.4597	−.44
$2^h\ 35^m$					

Name	Mag.	λ 1900	β	*log δ*	*c*
$2^h\ 37^m$					
θ Persei.....	4.0	53° 16.0′	+31° 37′	1.4040	—0.32
γ Ceti	3.3	38 02.3	—12 00	1.4641	— .44
π Ceti	4.0	32 21.2	—28 15	1.4186	— .41
μ Ceti	4.0	40 32.2	— 5 34	1.4716	— .43
η Persei.....	3.6	57 18.5	+37 28	1.3733	— .28
$2^h\ 44^m$					
41 Arietis	3.8	46 48.4	+10 27	1.4665	— .40
τ Persei.....	4.0	56 31.1	+34 22	1.3904	— .29
η Eridani	3.0	37 20.9	—24 33	1.4326	— .41
α Ceti.....	2.3	42 55.4	—12 35	1.4631	— .41
γ Persei.....	3.0	58 37.6	+34 31	1.3896	— .28
$2^h\ 58^m$					
ρ Persei.....	Var.	53 30.8	+20 34	1.4451	— .35
β Persei.....	Var.	54 46.4	+20 25	1.4396	— .34
ι Persei.....	4.0	57 50.4	+30 38	1.4084	— .30
12 Eridani	3.3	33 11.0	—44 43	1.3253	— .33
α Persei.....	2.0	60 41.2	+30 07	1.4107	— .28
$3^h\ 18^m$					
o Tauri	3.6	49 46.1	— 9 20	1.4679	— .39
ξ Tauri	3.6	50 30.8	— 8 48	1.4686	— .38
f Tauri	4.0	52 11.7	— 5 55	1.4714	— .38
ϵ Eridani	3.0	46 47.8	—27 44	1.4207	— .36
δ Persei.....	3.1	63 24.5	+27 18	1.4225	— .27
$3^h\ 37^m$					
o Persei.....	4.0	59 44.8	+12 10	1.4638	— .32
ν Persei.....	4.0	62 25.8	+22 08	1.4405	— .29
δ Eridani	3.0	49 27.5	—28 43	1.4167	— .34
17 Tauri	4.1	58 00.9	+ 4 11	1.4725	— .34
η Tauri	3.0	58 35.8	+ 4 03	1.4726	— .34
$3^h\ 42^m$					
τ^6 Eridani.....	4.0	45 58.0	—41 53	1.3456	— .31
27 Tauri	4.0	58 57.5′	+ 3 54	1.4727	— .34
ζ Persei	3.0	61 43.7	+11 19	1.4652	— .31
ϵ Persei	3.3	64 16.9	+19 06	1.4491	— .29
ξ Persei	4.0	63 34.6	+14 56	1.4588	— .30
$3^h\ 53^m$					
γ Eridani.....	3.0	52 27.9	—33 12	1.3963	— .31
λ Tauri	Var.	59 14.2	— 7 58	1.4695	— .33
ν Tauri	4.0	58 31.2	—14 28	1.4597	— .33
c Persei	4.0	68 06.1	+26 14	1.4265	— .25
o^2 Eridani.....	4.6	58 52.4	—28 21	1.4182	— .30
$4^h\ 12^m$					
γ Tauri	4.0	64 24.3	— 5 44	1.4715	— .30
δ Tauri	4.0	65 28.3	— 3 59	1.4727	— .29
ϵ Tauri	3.6	67 03.9	— 2 35	1.4733	— .28
θ^2 Tauri	3.6	66 33.7	— 5 51	1.4714	— .28
α Tauri	1.0	68 23.5	— 5 28	1.4717	— .27
$4^h 31^m$					

Name	Mag.	λ 1900	β	log b	c
$4^h\ 31^m$					
ν Eridani	4.0	65°25.0′	−25°08′	1.4305	−0.26
53 Eridani	4.0	63 51.4	−36 01	1.3816	− 25
μ Eridani.....	3.6	67 56.2	−25 23	1.4296	− 25
π^3 Orionis.....	3.5	70 30.7	−15 24	1.4578	− 25
π^5 Orionis	4.0	71 05.6	−20 01	1.4466	− 24
$4^h\ 50^m$					
ι Aurigae	3.0	75 14.6	+10 27	1.4665	− 22
10 Camelopardis ..	4 0	79 52.4	+37 25	1.3737	− 14
ε Aurigae	Var.	77 26.7	+20 56	1.4441	− 19
ζ Aurigae	4.0	77 14.2	+18 11	1.4514	− 19
η Aurigae	3.6	78 03.0	+18 16	1.4512	− 19
$5^h\ 0^m$					
ε Leporis	3.5	70 39.3	−44 58	1.3234	− 18
β Eridani	3.0	73 52.9	−27 52	1.4201	− 20
λ Eridani	4.0	73 48.9	−31 33	1.4042	− 20
μ Leporis	3.3	73 59.6	−39 04	1.3638	− 18
α Aurigae.....	1.0	80 27.6	+22 52	1.4382	− 16
$5^h\ 10^m$					
β Orionis	1.0	75 26.0	−31 08	1.4061	− 19
τ Orionis	4.0	76 27.0	−29 51	1.4119	− 19
η Orionis	3.3	78 45.6	−25 33	1.4290	− 17
γ Orionis.	2.0	79 33.0	−16 50	1.4547	− 18
β Tauri	2.0	81 10.7	+ 5 23	1.4718	− 17
$5^h\ 20^m$					
β Leporis	3.2	78 16.5	−43 55	1.3312	− 14
δ Orionis	Var.	80 57.9	−23 34	1.4359	− 16
α Leporis	3.0	79 59.0	−41 04	1.3510	− 14
λ Orionis	3.5	82 18.6	−13 23	1.4618	− 16
ι Orionis	3.1	81 36.1	−29 13	1.4146	− 15
$5^h\ 31^m$					
ε Orionis	2.0	82 04.0	−24 31	1.4327	− 15
ζ Tauri	3.3	83 23.3	− 2 12	1.4734	− 15
σ Orionis	3.7	82 41.9	−25 57	1.4275	− 14
ζ Orionis	2.0	83 17.1	−25 18	1.4299	− 14
γ Leporis	3.9	83 27.7	−45 49	1.3169	− 11
$5^h\ 41^m$					
ζ Leporis.....	3.6	84 35.4	−38 14	1.3689	− 11
κ Orionis	2.6	85 00.1	−33 05	1.3969	− 12
ν Aurigae.....	4.0	86 53.3	+15 42	1.4572	− 12
δ Leporis.....	4.0	85 45.8	−44 17	1.3285	− 10
α Orionis	Var.	87 21.5	−16 02	1.4565	− 11
$5^h\ 50^m$					
δ Aurigae	4.1	88 31.1	+30 50	1.4075	− 10
η Leporis	3.6	87 30.3	−37 37	1.3724	− 09
β Aurigae.....	2 0	88 30.9	+21 30	1.4423	− 10
θ Aurigae.....	3.0	88 32.7	+13 46	1.4611	− 11
η Geminorum	Var.	92 02.5	− 0 54	1.4736	− 07
$6^h\ 9^m$					

Name	Mag.	λ 1900	β	*log b*	*c*
$6^h\ 16^m$					
μ Geminorum	3.0	93°54.2′	— 0°50′	1.4737	—0.06
β Canis Maj.....	2.6	95 47.7	—41 16	1.3497	— .04
γ Geminorum	2.3	97 42.4	— 6 45	1.4707	— .03
S Monocerotis....	Var.	98 58.2	—13 11	1.4621	— .02
ϵ Geminorum	3.3	98 32.4	+ 2 04	1.4734	— .02
$6^h\ 39^m$					
ξ Geminorum	3.6	99 49.0	—10 07	1.4669	— .01
α Canis Maj.....	1.0	102 42.3	—39 35	1.3606	+ .01
θ Geminorum	3.3	99 43.6	+11 00	1.4656	— .01
ϵ Canis Maj.....	1.6	109 22.4	—51 22	1.2691	+ .04
ζ Geminorum	Var.	103 35.6	— 2 03	1.4734	+ .02
$7^h\ 0^m$					
δ Canis Maj.....	2.0	112 00.4	—48 28	1.2952	+ .06
λ Geminorum	3.8	107 23.1	— 5 39	1.4716	+ .05
δ Geminorum	3.3	107 07.4	— 0 11	1.4737	+ .05
R Canis Maj.....	Var.	113 05.8	—38 10	1.3692	+ .08
ι Geminorum	4.0	107 33.8	+ 5 45	1.4715	+ .05
$7^h\ 20^m$					
η Canis Maj.....	2.4	118 09.1	—50 37	1.2761	+ .09
β Canis Min.....	3.0	110 47.9	—13 30	1.4615	+ .08
α Geminorum	2.0	108 50.8	+10 06	1.4669	+ .06
α Canis Min.....	1.0	114 24.4	—16 00	1.4566	+ .11
κ Geminorum	3.6	112 16.2	+ 3 04	1.4731	+ .09
$7^h\ 39^m$					
β Geminorum	1.3	111 50.2	+ 6 41	1.4708	+ .09
ι Navis	3.0	130 00.3	—43 17	1.3358	+ .17
β Cancri	3.6	122 51.8	—10 18	1.4667	+ .18
30 Monocerotis....	3.6	128 27.7	—22 27	1.4395	+ .21
o Ursae Maj	3.3	111 35.9	+40 14	1.3565	+ .07
$8^h\ 30^m$					
δ Cancri	4.0	127 19.5	+ 0 05	1.4737	+ .22
ϵ Hydrae	3.3	130 57.3	—11 07	1.4655	+ .24
ζ Hydrae	3.3	133 11.0	—10 59	1.4657	+ .26
ι Ursae Maj	3.0	121 24.6	+29 35	1.4131	+ .15
α Cancri	4.0	132 14.7	— 5 05	1.4720	+ .26
$8^h\ 54^m$					
10 Ursae Maj	4.0	123 53.1	+23 43	1.4354	+ .18
κ Ursae Maj	3.3	122 32.2	+28 58	1.4157	+ .16
θ Hydrae.....	4.0	138 53.3	—13 03	1.4624	+ .30
40 Lyncis	3.3	130 26.9	+17 58	1.4520	+ .23
α Hydrae.....	2.0	145 53.3	—22 23	1.4397	+ .32
$9^h\ 23^m$					
h Ursae Maj	3.3	119 24.9	+45 09	1.3220	+ .11
θ Ursae Maj	3.0	125 53.0	+34 55	1.3875	+ .17
o Leonis	3.6	142 51.3	— 3 45	1.4728	+ .33
ϵ Leonis	3.0	139 18.5	+ 9 43	1.4674	+ .30
υ Ursae Maj	3.6	124 52.3	+42 39	1.3403	+ .15
$9^h\ 44^m$					

Name	Mag.	λ 1900	β	log b	c
$9^h\ 47^m$					
μ Leonis	4.0	140°02.1′	+ 12°21′	1.4636	+0.30
η Leonis	3.3	146 30.5	+ 4 52	1.4721	+ .35
α Leonis	1.3	148 26.4	+ 0 28	1.4737	+ 37
λ Hydrae.....	4.0	157 58.8	− 22 01	1.4408	+ .39
λ Ursae Maj.....	3.3	138 09.1	+ 29 53	1.4118	+ 26
$10^h\ 11^m$					
ζ Leonis	3.0	146 10.1	+ 11 52	1.4643	+ .34
γ Leonis	2.0	148 12.4	+ 8 49	1.4686	+ .36
μ Ursae Maj.....	3.0	139 50.2	+ 29 00	1.4156	+ .27
μ Hydrae.....	4.0	163 38.9	− 24 40	1.4321	+ .40
ρ Leonis	4.0	154 59.5	+ 0 09	1.4737	+ .40
$10^h\ 30^m$					
U Hydrae.....	Var.	164 53.8	− 20 24	1.4456	+ .42
ν Hydrae.....	3.3	168 58.5	− 21 48	1.4415	+ .43
46 Leonis Min	4.0	149 28.1	+ 24 56	1.4312	+ .34
β Ursae Maj.....	2.3	138 01.5	+ 45 07	1.3223	+ .21
α Ursae Maj.....	2.0	133 47.5	+ 49 41	1.2847	+ .17
$11^h\ 0^m$					
ψ Ursae Maj.....	3.1	147 24.6	+ 35 32	1.3842	+ .29
β Crateris	4.0	177 09.6	− 25 38	1.4287	+ .44
δ Leonis	2.3	159 54.7	+ 14 20	1.4600	+ .41
θ Leonis	3.3	162 01.5	+ 9 41	1.4675	+ .43
ξ Ursae Maj.	3.8	155 56.4	+ 24 44	1.4319	+ .37
$11^h\ 13^m$					
ν Ursae Maj.....	3.3	155 15.1	+ 26 09	1.4268	+ .36
δ Crateris	3.3	175 18.0	− 17 35	1.4529	+ .46
ι Leonis	4.0	166 09.7	+ 6 06	1.4712	+ .45
γ Crateris	4.0	177 50.7	− 19 40	1.4476	+ .46
λ Draconis	3.3	128 55.4	+ 57 14	1.2071	+ .13
$11^h\ 27^m$					
ξ Hydrae.....	4.0	186 36.3	− 31 35	1.4041	+ .42
χ Ursae Maj.....	3.8	152 15.5	+ 41 33	1.3479	+ .29
β Leonis	2.0	170 13.8	+ 12 17	1.4637	+ .45
β Virginis	3.3	175 44.7	+ 0 41	1.4737	+ .48
1830 Groombridge...	6.5	165 09.0	+ 36 00	1.3817	+ .36
$11^h\ 48^m$					
γ Ursae Maj.....	2.3	149 03.9	+ 47 08	1.3064	+ .25
o Virginis	4.0	176 18.1	+ 8 32	1.4689	+ .48
ε Corvi	3.0	190 16.6	− 19 40	1.4476	+ .47
δ Ursae Maj.....	3.4	149 39.0	+ 51 39	1.2664	+ .23
γ Corvi.....	2.0	189 20.2	− 14 30	1.4597	+ .48
$12^h\ 12^m$					
η Virginis	3.3	183 26.2	+ 1 22	1.4736	+ .49
δ Corvi	2.3	192 03.7	− 12 11	1.4638	+ .49
β Corvi	2.3	195 58.5	− 18 02	1.4518	+ .47
κ Draconis	3.3	134 50,6	+ 61 45	1.1488	+ .13
γ Virginis	3.0	188 45.5	+ 2 48	1.4732	+ .50
$12^h\ 37^m$					

Name	Mag.	λ 1900	β	*log b*	*c*
$12^h\ 49^m$					
ε Ursae Maj.	2.0	157°30.9′	+54°19′	1.2397	+0.24
δ Virginis	3.0	190 04.3	+ 8 38	1.4688	+ .49
12 Canum ven.....	2.9	173 09.8	+40 07	1.3572	+ .36
ε Virginis	2.6	188 32.9	+16 13	1.4561	+ .48
γ Hydrae.....	3.2	205 37.3	−13 44	1.4611	+ .47
$13^h\ 15^m$					
ζ Ursae Maj.....	2.1	164 16.7	+56 22	1.2170	+ .25
α Virginis	1.0	202 26.8	− 2 03	1.4734	+ .49
ζ Virginis	3.3	200 44.6	+ 8 39	1.4687	+ .49
η Ursae Maj.....	2.0	175 31.3	+54 23	1.2388	+ .28
η Boötis.	3.0	197 55.9	+28 06	1.4193	+ .44
$13^h\ 50^m$					
τ Virginis	4.0	206 20.9	+13 05	1.4623	+ .47
α Draconis.....	3.3	156 02.1	+66 22	1.0769	+ .16
ι Virginis	4.0	212 23.7	+ 7 14	1.4703	+ .46
α Boötis.	1.0	202 50.3	+30 48	1.4076	+ .42
λ Boötis.	4.0	185 33.6	+54 39	1.2361	+ .29
$14^h\ 20^m$					
θ Boötis.....	3.8	181 11.2	+60 07	1.1710	+ .24
ρ Boötis.....	3.6	201 23.0	+42 27	1.3416	+ .36
γ Boötis.....	2.9	196 15.5	+49 33	1.2858	+ .32
ζ Boötis.....	3.3	211 38.0	+27 53	1.4201	+ .41
μ Virginis	4.0	218 43.6	+ 9 42	1.4675	+ .44
$14^h\ 40^m$					
109 Virginis	3.6	217 07.2	+17 07	1.4540	+ .43
α Librae.....	2.3	223 41.3	+ 0 21	1.4737	+ .42
ξ Boötis	5.0	212 08.3	+33 47	1.3934	+ .39
β Ursae Min.....	2.0	131 53.6	+72 59	0.9402	+ .07
δ Librae	Var.	223 53.2	+ 8 16	1.4692	+ .42
$14^h\ 57^m$					
β Boötis	3.0	202 50.4	+54 10	1.2412	+ .29
γ Scorpii.....	3.4	229 17.6	− 7 38	1.4698	+ .39
δ Boötis.....	3.0	211 44.6	+48 59	1.2908	+ .31
β Librae.....	2.0	227 58.6	+ 8 31	1.4689	+ .40
μ Boötis.....	3.8	211 46.5	+53 26	1.2488	+ .28
$15^h\ 21^m$					
γ Ursae Min.....	3.0	140 09.6	+75 14	0.8800	+ .08
ι Draconis.....	3.0	183 30.9	+71 06	0.9842	+ .16
β Coronae bor....	3.8	217 43.2	+46 04	1.3150	+ .31
θ Coronae bor....	4.0	218 02.6	+48 34	1.2944	+ .29
α Coronae bor....	2.0	220 53.1	+44 20	1.3282	+ .31
$15^h\ 35^m$					
γ Coronae bor. ...	3.8	223 28.3	+44 31	1.3268	+ .30
α Serpentis.....	2.3	230 40.2	+25 31	1.4291	+ .35
β Serpentis.....	3.3	228 32.6	+34 20	1.3905	+ .33
κ Serpentis.....	4.0	228 22.4	+37 07	1.3754	+ .32
μ Serpentis.....	3.3	234 32.6	+16 15	1.4560	+ .35
$15^h\ 45^m$					

Name	Mag.	λ_{1900}	β	$\log b$	c
$15^h\ 45^m$					
ϵ Serpentis.....	3.3	232°55.7′	+24°01′	1.4344	+0.34
γ Serpentis.....	3.6	231 21.5	+35 14	1.3858	+ .31
ϵ Coronae bor....	4.0	227 42.9	+46 05	1.3148	+ .28
δ Scorpii	2.3	241 10.5	— 1 58	1.4735	+ .32
β Scorpii	2.0	241 47.6	+ 1 01	1.4737	+ .32
$16^h\ 0^m$					
θ Draconis	3.6	195 16.5	+74 27	0.9022	+ .13
ϕ Herculis	4.0	220 13.0	+63 47	1.1188	+ .19
δ Ophiuchi	3.0	240 54.3	+17 15	1.4537	+ .31
ϵ Ophiuchi	3.3	242 00.6	+16 27	1.4556	+ .30
τ Herculis	3.3	222 58.2	+65 50	1.0857	+ .17
$16^h\ 17^m$					
γ Herculis	3.1	237 48.9	+40 01	1.3578	+ .26
η Draconis	2.6	193 02.5	+78 27	0.7754	+ .10
α Scorpii	1.3	248 22.0	— 4 33	1.4723	+ .27
λ Ophiuchi	3.7	244 11.7	+23 35	1.4358	+ .28
β Herculis	2.3	239 41.5	+42 43	1.3398	+ .24
$16^h\ 30^m$					
ζ Ophiuchi.....	2.6	247 49.9	+11 24	1.4650	+ .27
ζ Herculis	2.6	240 04.9	+53 07	1.2520	+ .20
η Herculis	3.1	237 22.6	+60 18	1.1686	+ .17
κ Ophiuchi	3.3	250 25.9	+31 51	1.4028	+ .22
ϵ Herculis	3.3	246 55.7	+53 16	1.2505	+ .17
$17^h\ 0^m$					
η Ophiuchi	2.3	256 34.3	+ 7 12	1.4703	+ .21
ζ Draconis	3.0	181 50.6	+84 46	0.4340	+ .04
α Herculis.....	Var.	254 45.2	+37 18	1.3743	+ .18
δ Herculis	3.0	253 21.9	+47 42	1.3018	+ .16
U Ophiuchi	Var.	256 40.5	+24 13	1.4337	+ .19
$17^h\ 11^m$					
π Herculis	3.1	250 40.0	+59 34	1.1783	+ .13
θ Ophiuchi	3.4	259 59.9	— 1 50	1.4735	+ .18
β Draconis	2.6	250 33.7	+75 17	0.8783	+ .06
α Ophiuchi	2.0	261 02.8	+35 52	1.3824	+ .14
ξ Serpentis	3.6	263 09.0	+ 7 57	1.4695	+ .15
$17^h\ 35^m$					
ι Herculis	3.3	258 29.4	+69 17	1.0224	+ .07
β Ophiuchi	3.0	263 56.5	+27 57	1.4198	+ .13
X Sagitarii.....	Var.	265 50.7	— 4 24	1.4724	+ .13
μ Herculis	3.3	263 50.4	+51 08	1.2712	+ .09
γ Ophiuchi	3.6	265 14.2	+26 08	1.4269	+ .12
$17^h\ 50^m$					
ξ Draconis	3.3	263 20.9	+80 17	0.7006	+ .03
θ Herculis	4.0	267 04.9	+60 42	1.1633	+ .06
ν Ophiuchi	3.6	268 21.4	+13 41	1.4612	+ .11
ξ Herculis.....	3.6	267 47.8	+52 42	1.2561	+ .07
γ Draconis	2.3	266 34.5	+74 56	0.8885	+ .03
$17^h\ 55^m$					

Name	Mag.	λ 1900	β	*log b*	*c*
$17^h\ 55^m$					
67 Ophiuchi	4.0	268° 47.1′	+ 26° 24′	1.4259	+0.10
W Sagitarii.....	Var.	269 42.0	— 6 08	1.4712	+ .10
γ Sagitarii	3.3	269 52.0	— 6 58	1.4705	+ .10
70 Ophiuchi.....	4.7	270 06.7	+ 25 59	1.4275	+ .09
72 Ophiuchi.....	3.3	270 46.0	+ 33 00	1.3973	+ .08
$18^h\ 3^m$					
ο Herculis	3.8	271 18.1	+ 52 12	1.2611	+ .05
μ Sagitarii	4.0	271 49.0	+ 2 21	1.4733	+ .08
Υ Sagitarii	Var.	273 40.6	+ 4 30	1.4724	+ .07
η Serpentis.....	3.0	274 18.0	+ 20 28	1.4454	+ .06
109 Herculis	4.0	276 23.8	+ 45 04	1.3227	+ .03
$18^h\ 20^m$					
χ Draconis.....	3.8	74 42.4	+ 83 33	0.5242	— .03
α Lyrae	1.0	283 54.9	+ 61 44	1.1490	— .01
110 Herculis.....	4.0	283 23.3	+ 43 25	1.3348	— .01
β Lyrae	Var.	287 29.8	+ 56 00	1.2213	— .03
σ Sagitarii.....	2.3	280 59.3	— 3 26	1.4729	.00
$18^h\ 50^m$					
R Lyrae.....	Var.	293 50.7	+ 66 11	1.0798	— .04
ε Aquilae	4.0	286 52.3	+ 37 35	1.3727	— .04
γ Lyrae	3.3	290 32.1	+ 55 02	1.2320	— .05
ζ Aquilae	3.0	288 24.3	+ 36 12	1.3805	— .05
λ Aquilae	3 1	285 56.3	+ 17 35	1.4529	— .04
$19^h\ 2^m$					
π Sagitarii	3.1	284 51.3	+ 1 28	1.4736	— .03
δ Draconis.....	3.0	15 50.7	+ 82 53	0.5667	— .06
κ Cygni	4.0	313 32.8	+ 73 48	0.9190	— .07
δ Aquilae	3.3	292 14.2	+ 24 50	1.4316	— .09
β Cygni	3.0	299 51.9	+ 48 59	1.2908	— .10
$19^h\ 30^m$					
σ Draconis	5.3	29 16.9	+ 80 55	0.6719	— .07
γ Aquilae	3.0	299 32.8	+ 31 15	1.4056	— .13
δ Cygni	2.8	314 52.2	+ 64 25	1.1089	— .12
δ Sagittae	4.0	301 59.8	+ 38 55	1.3647	— .14
α Aquilae	1.3	300 21.9	+ 29 19	1.4142	— .14
$19^h\ 47^m$					
η Aquilae	Var.	299 02.4	+ 21 32	1.4423	— .14
ε Draconis	3.8	31 20.4	+ 79 29	0.7349	— .09
β Aquilae	4.0	301 02.0	+ 26 41	1.4248	— .15
γ Sagittae	3.6	305 39.2	+ 39 12	1.3630	— .16
θ Aquilae.....	3.0	303 31.1	+ 18 44	1.4500	— .18
$20^h\ 10^m$					
a^2 Capricorni	3.3	302 27.7	+ 6 57	1.4705	— .18
β Capricorni	3.0	302 39.0	+ 4 36	1.4723	— .18
γ Cygni	2.4	323 27.8	+ 57 08	1.2083	— .18
θ Cephei.....	4.0	3 30.0	+ 73 56	0.9158	— .14
ε Delphini.....	4.0	312 40 2	+ 29 05	1.4152	— .23
$20^h\ 29^m$					

Name	Mag.	λ 1900	β	log b	c
$20^h\ 32^m$					
β Delphini.....	3.3	314° 56.9′	+31° 56′	1.4025	— 0.23
α Delphini.....	3.6	315 59.3	+33 02	1.3972	— .24
α Cygni.....	1.6	333 57.3	+59 55	1.1738	— .20
δ Delphini.....	4.0	316 43.7	+31 57	1.4023	— .24
γ Delphini.....	4.0	317 58.9	+32 43	1.3987	— .25
$20^h\ 42^m$					
ε Cygni.....	2.6	326 20.6	+49 25	1.2869	— .23
ε Aquarii.....	3.6	310 19.6	+ 8 06	1.4694	— .24
η Cephei.....	3.6	3 15.0	+71 46	0.9691	— .15
ν Cygni.....	4.0	334 45.6	+54 55	1.2332	— .23
ξ Cygni.....	4.0	339 25.6	+56 35	1.2146	— .23
$21^h\ 2^m$					
61² Cygni.....	5.7	335 20.5	+51 53	1.2642	— .25
ζ Cygni.....	3.0	331 39.5	+43 42	1.3328	— .28
τ Cygni.....	4.0	337 12.7	+50 33	1.2768	— .26
α Aquilae.....	4.0	321 43.5	+20 08	1.4463	— .30
α Cephei.....	2.6	11 24.3	+68 55	1.0298	— .18
$21^h\ 20^m$					
β Aquarii.....	3.0	322 00.0	+ 8 38	1.4688	— .32
β Cephei.....	3.0	34 10.7	+71 09	0.9832	— .15
γ Capricorni.....	3.6	320 23.3	— 2 33	1.4733	— .31
ε Pegasi.....	2.3	330 29.6	+22 08	1.4405	— .35
κ Pegasi.....	4.0	337 32.7	+36 39	1.3781	— .33
$21^h\ 40^m$					
μ Cephei.....	Var.	8 20.3	+64 11	1.1126	— .22
δ Capricorni.....	3.0	322 08.5	— 2 35	1.4733	— .33
α Aquarii.....	3.0	331 57.5	+10 40	1.4661	— .38
ι Aquarii.....	4.0	327 19.3	— 2 04	1.4734	— .36
ι Pegasi.....	4.0	343 00.5	+34 16	1.3909	— .36
$22^h\ 4^m$					
θ Pegasi.....	3.3	335 26.0	+16 21	1.4558	— .39
ζ Cephei.....	3.4	12 35.3	+61 09	1.1573	— .24
γ Aquarii.....	3.4	335 19.0	+ 8 15	1.4692	— .40
ζ Aquarii.....	4.0	337 30.6	+ 8 51	1.4685	— .41
δ Cephei.....	Var.	16 14.0	+59 32	1.1787	— .25
$22^h\ 26^m$					
7 Lacertae.....	4.0	6 45.5	+53 18	1.2502	— .37
η Aquarii.....	3.8	339 00.4	+ 8 09	1.4693	— .42
ζ Pegasi.....	3.3	344 45.5	+17 41	1.4527	— .42
η Pegasi.....	3.0	354 19.6	+35 07	1.3865	— .39
λ Pegasi.....	4.0	351 39.9	+28 48	1.4164	— .41
$22^h\ 43^m$					
τ Aquarii.....	4.0	337 11.9	— 5 40	1.4716	— .41
μ Pegasi.....	4.0	352 59.6	+29 24	1.4139	— .41
ι Cephei.....	3.4	31 52.3	+62 37	1.1365	— .21
λ Aquarii.....	4.0	340 10.7	— 0 23	1.4737	— .43
δ Aquarii.....	3.0	337 28.6	— 8 11	1.4693	— .41
$22^h\ 50^m$					

Name	Mag.	λ 1900	β	*log b*	*c*
$22^h\ 52^m$					
α Piscis aus.....	1.3	332°27.1′	−21°07′	1.4435	−0.37
ο Andromedae ...	3.6	6 23.7	+43 45	1.3325	− .36
β Pegasi	Var.	357 58.7	+31 08	1.4061	− .41
α Pegasi	2.0	352 05.5	+19 25	1.4483	− .44
O^2 Aquarii.....	4.0	338 37.0	−14 29	1.4597	− .41
$23^h\ 5^m$					
3077 Bradley	6.0	22 16.8	+54 34	1.2369	− .28
γ Piscium	4.0	350 02.3	+ 7 17	1.4702	− .46
λ Andromedae ...	4.0	16 54.4	+43 47	1.3322	− .36
ι Andromedae ...	4.0	14 41.9	+41 02	1.3513	− .38
γ Cephei	3.3	58 42.1	+64 39	1.1053	− .14
$23^h\ 40^m$					
ω Piscium	4.0	1 11.1	+ 6 22	1.4710	− .49

PRECESSION IN LONGITUDE.

1901	+0.8′	1911	+ 9.2′	1921	+17.6′
1902	+1.7	1912	+10.1	1922	+18.4
1903	+2.5	1913	+10.9	1923	+19.3
1904	+3.4	1914	+11.7	1924	+20.1
1905	+4.2	1915	+12.6	1925	+20.9
1906	+5.0	1916	+13.4	1926	+21.8
1907	+5.9	1917	+14.2	1927	+22.6
1908	+6.7	1918	+15.1	1928	+23.5
1909	+7.5	1919	+15.9	1929	+24.3
1910	+8.4	1920	+16.8	1930	+25.1

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THEORY OF THE DETERMINATION OF THE ELEMENTS OF A PARABOLIC ORBIT FROM TWO OBSERVATIONS OF APPARENT POSITION, AND ONE OF THE MOTION IN THE LINE OF SIGHT.

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WITH the great light-gathering power of modern telescopes, and the many improvements in the construction and methods of application of the spectroscope, it may be expected that measurements of relative motion in the line of sight will be made with a very high degree of precision. If such measurements of relative motion can be secured for a body moving around the Sun, together with the necessary number of apparent positions, it is evident that these data may be made the basis for the determination of the elements of its orbit. The purpose of this paper is to set forth the theory of the computation of the elements, supposing that the orbit is a parabola, and that at two epochs observations of the apparent position have been made, and that at the second epoch the motion in the line of sight has been observed.

§ 1. FORMULATION OF THE PROBLEM.

Let λ, β, and ρ represent the longitude, latitude, and distance respectively of the comet, taking the center of the Earth as origin. Let the derivatives of these quantities with respect to the time be represented by the same letters with primes. Suppose the observations are made at the two epochs, t_1 and t_2.

The observed quantities λ_1, λ_2, β_1, β_2, and ρ_2' depend upon the elements of the unknown orbit, hence we may write the following equations:

$$\left.\begin{aligned} \lambda_1 &= \theta_1 \text{ (elements)}, \\ \lambda_2 &= \theta_2 (\ldots\ldots), \\ \beta_1 &= \phi_1 (\ldots\ldots), \\ \beta_2 &= \phi_2 (\ldots\ldots), \\ \rho_2' &= \psi_2 (\ldots\ldots). \end{aligned}\right\} \quad (1)$$

These five equations involve the five elements as unknowns, and when solved, furnish the solution to the problem. They are, however, transcendental, and so exceedingly involved that a direct solution of them would present almost insurmountable difficulties. This suggests the alternative of first determining *intermediate quantities* from which the elements can be found. As such, it may be mentioned that one absolute position with reference to the Sun, and the velocity and direction of motion, are sufficient for the unique determination of the elements. If one desired to determine these intermediate quantities it would only be necessary to find *one* geocentric distance and the velocity and direction of motion. That is, there would be but *four* unknowns involved, instead of the five elements, and the problem would be reduced to the solution of *four* simultaneous equations.

As another set of intermediate quantities, the six coördinates defining two positions of the comet with respect to the Sun, and the interval of time required for the comet to move from one to the other, might be used. The interval of time is known, and the only unknowns in the two positions are the two geocentric distances. Therefore, $\lambda_1 \ldots \rho_2'$ may be considered as functions of the two unknowns, ρ_1 and ρ_2. In this case then, which is clearly the simplest possible, it is only necessary to find and solve two independent equations involving ρ_1, ρ_2 and known quantities.

Neglecting perturbations, the work must be consistent with and involve the following fundamental theorems of parabolic motion.

Theorem I. The motion of the comet is in a plane passing through the center of the Sun.

Theorem II. The areas swept over by the radius vector from the Sun are proportional to the intervals of time in which they are described.

Theorem III. The motion is in a parabola with the Sun at its focus.

It will be supposed that the effects of the Earth's motions upon the observed velocities in the line of sight have been

eliminated, so that ρ_2' represents the velocity in the direction of the Earth with respect to the Sun. The formulae for accomplishing this were given in the March number of the ASTROPHYSICAL JOURNAL, by Dr. Frank Schlesinger.

§ 2. THE TWO INDEPENDENT EQUATIONS INVOLVING ρ_1 AND ρ_2 AS UNKNOWNS.

Let x, y, and z represent the rectangular heliocentric coördinates, and r, l, and b the polar. Denote by s the chord joining the extremities of the radii r_1 and r_2. Let k represent the Gaussian constant. Then Euler's equation is

$$6k(t_2 - t_1) = (r_1 + r_2 + s)^{\frac{3}{2}} \mp (r_1 + r_2 - s)^{\frac{3}{2}}. \qquad (2)$$

The upper sign is to be used if the heliocentric motion in the interval of time t_2—t_1 is less than 180°, which will henceforth be supposed to be the case.

It will now be shown that (2) can be expressed in terms of ρ_1 and ρ_2 as unknowns. r_1, r_2, and s depend upon x_1, y_1, z_1, x_2, y_2, z_2, which are equal respectively to the differences of the corresponding geocentric coördinates of the comet and the Sun. The geocentric coördinates of the Sun are given in the Nautical Almanacs, and the geocentric coördinates of the comet involve as unknowns ρ_1 and ρ_2 alone. Therefore Euler's formula is one equation of the type sought.

As a consequence of Theorem I we may write the following equations:

$$\left.\begin{aligned} Ax_1 + By_1 + Cz_1 &= 0, \\ Ax_2 + By_2 + Cz_2 &= 0, \\ Ax_2' + By_2' + Cz_2' &= 0. \end{aligned}\right\} (3)$$

The constants A, B, and C, depend upon the position of the plane of the orbit, with respect to the plane of the ecliptic and the vernal equinox. Eliminating them we have,

$$\left.\begin{vmatrix} x_1, & y_1, & z_1, \\ x_2, & y_2, & z_2, \\ x_2', & y_2', & z_2', \end{vmatrix} = 0. \right\} (4)$$

The expansion of this determinant may be written in the three ways,

$$\left.\begin{aligned} x_1(y_2z_2' - z_2y_2') + x_2(z_1y_2' - y_1z_2') + x_2'(y_1z_2 - z_1y_2) &= 0, \\ y_1(z_2x_2' - x_2z_2') + y_2(x_1z_2' - z_1x_2') + y_2'(z_1x_2 - x_1z_2) &= 0, \\ z_1(x_2y_2' - y_2x_2') + z_2(y_1x_2' - x_1y_2') + z_2'(x_1y_2 - y_1x_2) &= 0. \end{aligned}\right\} (5)$$

As these equations are written they are not independent, but if by some new means the parentheses can be determined, they become independent and can be used for the elimination of any two unknowns. Let it be supposed for the moment that the parentheses have been computed. Then x_1, y_1, z_1, x_2, y_2, and z_2 depend upon ρ_1 and ρ_2 alone as unknowns, as was seen in the discussion of Euler's equation. x_2', y_2', and z_2' depend upon the direction of motion and velocity of the comet at the time t_2. They may be expressed in terms of ρ_2' and two other quantities. Eliminating these two unknown components of velocity, we have from (5) the desired relation between ρ_1 and ρ_2, which may be written,

$$F(\rho_1, \rho_2) = 0. \qquad (6)$$

Therefore (2) and (6) are the equations from which ρ_1 and ρ_2 may be found.

§ 3. THE COMPUTATION OF THE PARENTHESES.

Consider the parentheses in the first equation of (5). By the law of areas $(y_2z_2' - z_2y_2')$ is twice the projection, upon the yz plane, of the area described by the radius vector in a unit of time. Denoting the angles between the plane of the orbit and the three fundamental planes by i_{xy}, i_{yz} and i_{zx} respectively, we have,

$$(y_2z_2' - z_2y_2') = k\sqrt{p}\cos i_{yz}, \qquad (7)$$

where p is the parameter of the parabola. $(y_1z_2 - z_1y_2)$ is twice the projection, upon the yz plane, of the area between the radii r_1 and r_2, and the chord joining their extremities. Then we have,[1]

[1] OPPOLZER'S *Bahnbestimmung der Kometen und Planeten*, 1, p. 99, or WATSON'S *Theoretical Astronomy*, p. 176.

$$(y_1 z_2 - z_1 y_2) = k\sqrt{p}\,(t_2 - t_1)\left\{1 - \frac{k^2}{6}\frac{(t_2-t_1)^2}{r_1^3} + \cdots\right\}\cos i_{yz}. \quad (8)$$

It remains to consider the second parenthesis of (5), $(z_1 y_2' - y_1 z_2')$. Since the coördinates and velocities are functions of the time we may write,

$$\left.\begin{aligned} y_1' &= f(t_1), \\ y_2' &= f[t_1 + (t_2 - t_1)]. \end{aligned}\right\} \quad (9)$$

We shall suppose that the interval of time $t_2 - t_1$ is short, then expanding the second of (9) by Taylor's formula we have,

$$\left.\begin{aligned} y_2' &= f(t_1) + f'(t_1)(t_2 - t_1) + f''(t_1)\frac{(t_2-t_1)^2}{2} + \cdots, \text{ or,} \\ y_2' &= y_1' + y_1''(t_2 - t_1) + y_1'''\frac{(t_2-t_1)^2}{2} + \cdots. \end{aligned}\right\} \quad (10)$$

The development for z_2' and x_2' are obtained by writing in place of y, z, and x, respectively. Substituting these expressions in the parenthesis $(z_1 y_2' - y_1 z_2')$ we find,

$$(z_1 y_2' - y_1 z_2') = (z_1 y_1' - y_1 z_1') + t_2 - t_1)(z_1 y_1'' - y_1 z_1'') + \frac{(t_2-t_1)}{2}(z_1 y_1''' - y_1 z_1''') + \cdots. \quad (11)$$

From the fundamental equations of acceleration $(z_1 y_1'' - y_1 z_1'')$ is identically zero. It is found by differentiating $(z_1 y_1'' - y_1 z_1'') = 0$ that

$$(z_1 y_1''' - y_1 z_1''') = -(z_1' y_1'' - y_1' z_1'') = -\frac{k^2}{r_1^3}(z_1 y_1' - y_1 z_1'). \quad (12)$$

Then (11) becomes

$$(z_1 y_2' - y_1 z_2') = (z_1 y_1' - y_1 z_1')\left(1 - \frac{k_2}{2}\frac{(t_2-t_1)^2}{r_1^3} + \cdots\cdots\right). \quad (13)$$

As a consequence of the law of areas this reduces to

$$(z_1 y_2' - y_1 z_2') = -k\sqrt{p}\left\{1 - \frac{k^2}{2}\frac{(t_2-t_1)^2}{r_1^3} + \cdots\right\}\cos i_{yz}. \quad (14)$$

There is the question whether in any case the terms of higher orders, which we have neglected, could become sensible. We shall suppose that the interval of time between the observations is at the most only a few days. The radius r_1 will generally be

in the neighborhood of unity. The factor $\frac{k_g}{2} = 0.000148$. The next higher term than the ones written in (14) involves, in addition to the factor k^2, the factor $\frac{dr_1}{dt}$. It is easy to see that in every case this will be an extremely small fraction. In order that it might be equal to unity the radius vector would have to change a whole astronomical unit in a mean solar day. Therefore we are safe in neglecting terms of the third and higher orders.

In computing the parentheses by (8) and (14), sufficiently accurate results for the first approximation to the orbit will be obtained, owing to the small factor $\frac{k_g}{2}$, in taking r_1 equal to unity. The parentheses may now be regarded as computed and the equations (5) become independent. All unknowns, except ρ_1 and ρ_2, must now be eliminated from them in order to find (6) explicitly.

§ 4. SOLUTION OF THE TWO EQUATIONS INVOLVING ρ_1 AND ρ_2 AS UNKNOWNS.

Substituting (7), (8) and (14) in (5) we find

$$\left.\begin{aligned}
&x_1 - x_2\left\{1 - \frac{k^2}{2}\frac{(t_2-t_1)^2}{r_1^3} + \cdots\right\} \\
&\qquad + x_2'\left\{1 - \frac{k^2}{6}\frac{(t_2-t_1)^2}{r_1^3} + \cdots\right\}(t_2-t_1) = 0, \\
&\text{and similarly} \\
&y_1 - y_2\left\{1 - \frac{k^2}{2}\frac{(t_2-t_1)^2}{r_1^3} + \cdots\right\} \\
&\qquad + y_2'\left\{1 - \frac{k^2}{6}\frac{(t_2-t_1)^2}{r_1^3} + \cdots\right\}(t_2-t_1) = 0, \\
&z_1 - z_2\left\{1 - \frac{k^2}{2}\frac{(t_2-t_1)^2}{r_1^3} + \cdots\right\} \\
&\qquad + z_2'\left\{1 - \frac{k^2}{6}\frac{(t_2-t_1)^2}{r_1^3} + \cdots\right\}(t_2-t_1) = 0.
\end{aligned}\right\} \quad (15)$$

Let the latitude of the Sun be neglected, and denote the

geocentric polar coördinates by R and $\odot$. Then x, y, z, x', y', and z' may be expressed as follows:

$$\left.\begin{aligned}
x &= \rho \cos\beta \cos\lambda - R\cos\odot,\\
y &= \rho \cos\beta \sin\lambda - R\sin\odot,\\
z &= \rho \sin\beta,\\
x' &= \cos\beta\cos\lambda\,\rho' - \rho\cos\beta\sin\lambda\,\lambda' - \rho\sin\beta\cos\lambda\,\beta'\\
&\qquad - \cos\odot\,R' + R\sin\odot\,\odot',\\
y' &= \cos\beta\sin\lambda\,\rho' + \rho\cos\beta\cos\lambda\,\lambda' - \rho\sin\beta\sin\lambda\,\beta'\\
&\qquad - \sin\odot\,R' - R\cos\odot\,\odot',\\
z' &= \sin\beta\,\rho' + \rho\cos\beta\,\beta'.
\end{aligned}\right\} \quad (16)$$

Apply the proper subscripts and substitute these equations in (15). Then, for abbreviation, let

$$\left.\begin{aligned}
\mu &= -1 + \frac{k^2}{2}\frac{(t_2 - t_1)^2}{r_1^3} + \cdots\cdots,\\
\nu &= \left\{1 - \frac{k^2}{6}\frac{(t_2 - t_1)^2}{r_1^3} + \cdots\right\}(t_2 - t_1).\\
a_1 &= -\nu\rho_2\cos\beta_2\sin\lambda_2,\\
a_2 &= -\nu\rho_2\sin\beta_2\cos\lambda_2,\\
a_3 &= \rho_1\cos\beta_1\cos\lambda_1 - R_1\cos\odot_1 + \mu\,(\rho_2\cos\beta_2\cos\lambda_2 - R_2\\
&\quad \cos\odot_2) + \nu\,(\cos\beta_2\cos\lambda_2\,\rho_2' - \cos\odot_2\,R_2' + R_2\sin\odot_2\,\odot_2').\\
b_1 &= \nu\rho_2\cos\beta_2\cos\lambda_2,\\
b_2 &= -\nu\rho_2\sin\beta_2\sin\lambda_2,\\
b_3 &= \rho_1\cos\beta_1\sin\lambda_1 - R_1\sin\odot_1 + \mu\,(\rho_2\cos\beta_2\sin\lambda_2 -\\
&\qquad R_2\sin\odot_2) + \nu\,(\cos\beta_2\sin\lambda_2\,\rho_2' - \sin\odot_2\,R_2' -\\
&\qquad\qquad R_2\cos\odot_2\,\odot_2').\\
c_1 &= 0,\\
c_2 &= \nu\rho_2\cos\beta_2,\\
c_3 &= \rho_1\sin\beta_1 + \mu\,\rho_2\sin\beta_2 + \nu\sin\beta_2\,\rho_2'.
\end{aligned}\right\} \quad (17)$$

Then (15) becomes

$$\left.\begin{aligned}
a_1\lambda_2' + a_2\beta_2' + a_3 &= 0,\\
b_1\lambda_2' + b_2\beta_2' + b_3 &= 0,\\
c_1\lambda_2' + c_2\beta_2' + c_3 &= 0.
\end{aligned}\right\} \quad (18)$$

Eliminating λ_2' and β_2' we have

$$\left.\begin{vmatrix} a_1, a_2, a_3\\ b_1, b_2, b_3\\ 0, c_2, c_3 \end{vmatrix} = 0.\right\} \quad (19)$$

Expanding the determinant, and carrying out the computation in the original quantities, it is found, after rejecting common factors, that

$$\left.\begin{aligned}&\mu\,\rho_2+\rho_1\,\{\sin\,\beta_1\,\sin\,\beta_2+\cos\,\beta_1\,\cos\,\beta_2\,\cos\,(\lambda_2-\lambda_1)\}+\nu\,\rho_2'\\&-R_1\,\cos\,\beta_2\,\cos\,(\lambda_2-\odot_1)-\mu\,R_2\,\cos\,\beta_2\,\cos\,(\lambda_2-\odot_2)\\&-\nu\,\cos\,\beta_2\,\{\,R_2'\,\cos\,(\lambda_2-\odot_2)+R_2\,\sin\,(\lambda_2-\odot_2)\,\odot_2'\}=0.\end{aligned}\right\}\quad(20)$$

Let

$$\left.\begin{aligned}m=&-\frac{\nu}{\mu}\,\rho_2'+\frac{R_1'}{\mu}\cos\,\beta_2\cos\,(\lambda_2-\odot_1)+R_2\cos\,\beta_2\cos\,(\lambda_2-\odot_2)\\&+\frac{\nu}{\mu}\cos\,\beta_2\,(R_2'\,\cos\,(\lambda_2-\odot_2)+R_2\,\sin\,(\lambda_2-\odot_2)\,\odot_2'\,\}\,.\\M=&-\frac{1}{\mu}\,\{\,\sin\,\beta_1\,\sin\,\beta_2+\cos\,\beta_1\,\cos\,\beta_2\,\cos\,(\lambda_2-\lambda_1)\,\}\,.\end{aligned}\right\}\quad(21)$$

Then (20) becomes,

$$\rho_2=m+M\,\rho_1,\qquad(22)$$

where m and M are known quantities. As was shown, Euler's equation, (2), depends upon ρ_1 and ρ_2 alone as unknowns. Therefore, (2) and (22) may be used for the determination of the two geocentric distances.

The equation (20) has been purposely reduced to the same form as that which arises in the theory of parabolic orbits, based on observations of position alone. The quantities m and M are entirely different in this case, but, because of the form of the equations, the solution would not be different from that given in Oppolzer's *Bahnbestimmung der Kometen und Planeten*, I, pp. 303–307. Therefore, it is not necessary to consider it here.

When this work is carried out, two positions of the comet are known, and the elements can at once be derived by familiar processes. Thus, the formulæ and references given in this paper are sufficient for the determination of the first approximation to the parabolic elements of a comet's orbit, when the computer has at his disposal two observations of position, and one of the motion in the line of sight.

THE UNIVERSITY OF CHICAGO,
May 20, 1899.

THE INFLUENCE OF THE PURKINJE PHENOMENON ON OBSERVATIONS OF FAINT SPECTRA.

By W. W. CAMPBELL.

THOSE who are familiar with the details of the spectrum observations of the Orion Nebula will, on reflection, easily convince themselves that the Purkinje phenomenon has very little to do with estimates of the relative intensities of the three principal lines. There remains the equally important question as to whether it enters the problem to any extent whatsoever.

This phenomenon seems to be an organic color effect, arising from the observer's effort to compare, numerically, two unlike objects. Professor Runge's laboratory observations showed a small Purkinje effect when he compared spectral regions of wavelengths λλ 4862 and 5007; but for the regions λλ 4862 and 4959, an effect which was at first apparent was later reduced to zero. Professor Runge's experiments were based upon light which was initially of "medium brightness;" but his published account shows that he was fully aware of the one important condition essential to a laboratory solution of this problem, viz., that the initial intensities of the artificial lines should equal those of the nebular lines.

Now the absolute brightness of the nebular lines is surprisingly low. [For an approximate measure of the intensity of the *Hβ* nebular line, see my paper in the May number of this JOURNAL.] The nebular lines are so faint that many—and I think all—observers are unable to distinguish between their colors. They lie so near the limit of color perception that some observers do not see them as blue-green, but as gray. In this and similar cases, does the Purkinje effect enter at all? Great numbers of observations are made under these conditions and it is important that the question should be settled.

In 1889, Hering and Hillebrand observed that the distribution of brightness in a faint colorless spectrum, as viewed by normal eyes, is identical with the distribution of brightness in a spectrum of *every* degree of intensity, as seen by totally color-blind observers. [*Sitzungsb. d. Wien. Akad.*, 1889; *Wied. Ann.*, **62**, 17.]

About the same time, Professor A. König observed "dass die Vertheilung der Helligkeit im Spectrum in einzelnen Fällen auch dann ungeändert bleibt, wenn durch peripher oder central gelegene pathologische Processe die eigentliche Farbenempfindung völlig verloren geht und nur die Empfindungsreihe Schwarz-Grau-Weiss bestehen bleibt." [*Wied. Ann.*, **45**, 607, 1892.]

These observations demonstrated that the Purkinje phenomenon is dependent upon *color-perception;* and that its effect upon observations of the Orion Nebula made by those who see the lines as gray is absolutely zero.

There remains the case of those who are able to perceive that the lines have a blue-green tinge, but who cannot distinguish *differences* in their colors. If the Purkinje phenomenon is a differential-color effect—and so it seems to be—this case would be very closely related to the preceding one in which the effect is zero.

Even if an observer were just able to distinguish differences in the colors of the lines—which I am strongly inclined to doubt—the observations of König and others clearly indicate that the effect would be exceedingly slight, and safely negligible.

The critic who assumed that a twenty-five to thirtyfold observed variation "is nothing more nor less than the Purkinje phenomenon," wrote that "Although the lines in question differ but slightly in wave-length, their brightness is so close to the limit of visibility that great differences in relative intensity . . . . may be produced." [This JOURNAL, **7**, 238.] It appears to me that this case of lines situated in the blue-green, which differ but slightly in wave-length, and whose brightness is so close to the limit of visibility, is precisely the case where the principles of color-vision would have justified the prediction of exceedingly

slight Purkinje effects. In this connection, it may be said that the observations by Hering, Hillebrand and König, bearing on this question, are among the most important in the whole domain of Physiological Optics.

LICK OBSERVATORY,
May 12, 1899.

RESULTS OF THE PHOTOGRAPHIC OBSERVATIONS OF THE LEONIDS, NOVEMBER 14–15, 1898, AT THE YALE OBSERVATORY.

By W. L. ELKIN.

AS STATED in a previous communication (this JOURNAL, **9**, 20) during twelve hours exposure on the nights of November 14 and 15, 1898, sixteen meteor trails were photographed at our two stations. Of these, eight proved to be Leonids, and eight meteors from other radiants. Seven of the Leonid trails have now been fully measured and discussed, one proving too short and at too great a distance from the radiant to be of any value, and a summary of the results is herewith presented.

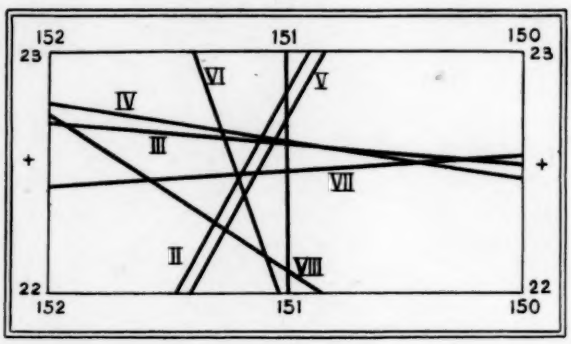
Five of the Leonid trails were secured with the apparatus at the Observatory, driven by clockwork, and the measurement of these plates presents no difficulty. The perpendicular distances of the track from all known stars within about 30′ were measured and the most probable track determined by least squares, the track being determined by the R. A. of its intersection with, and by its inclination to the equator, denoted respectively by "N" and "I."

For the plates taken at the Hamden station where the stars were allowed to trail for ten minutes, the perpendicular distances of the meteor tracks from the two ends of the star trail were measured. These two ends may be considered as two stars of the same declination as the star producing the trail, but with right ascensions differing from that of this star by t and $(10 - t)$ minutes, where t is the time between the moment of the meteor's appearance and that when the star began to trail.

The following table gives the results thus derived, the tracks being referred to the mean places of the stars for 1875.0.

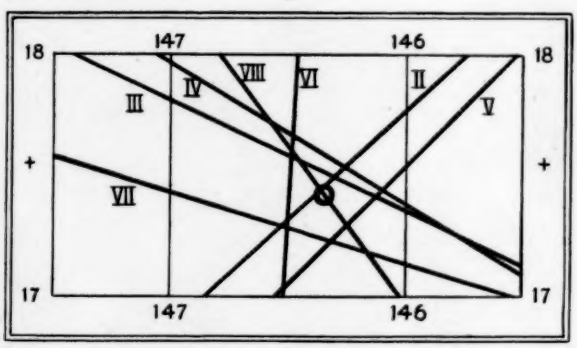
No.		Greenwich M. Time hr.	min.	N	I	Station
I.	Nov. 14	18	21.4	(Too short and distant)		Y
II.	" 14	18	15.7	341°46.3′	65°37.4′	Y
III.	" 14	20	56.9	74 26.8	23 16.4	Y
IV.	" 14	20	56.9	85 57.7	24 48.0	H
V.	" 14	20	36.8	342 17.9	65 4.8	Y
VI.	" 14	19	51.6	144 14.4	73 55.1	H
VII.	" 14	20	49.4	51 14.9	22 53.5	Y
VIII.	" 15	18	21.4	120 2.5	37 51.5	Y

A.



APPARENT RADIANT.

B



TRUE RADIANT. LONG. AND LAT.

I have corrected the prolongation of these tracks in the immediate neighborhood of the apparent radiant for the effect of the Earth's attraction and diurnal rotation, using for the

meteors the velocity derived from the period of 33.25 years. The apparent tracks thus derived are shown where they intersect near the radiant in Fig. A, and the apparent radiant might be derived from them at once. But I have thought it of more interest to determine the tracks which result from a combination of these apparent tracks with the orbital motion of the Earth, and Fig. B shows these tracks in the vicinity of the true direction of the radiant in longitude and latitude. The point for which the sum of the squares of its distance from the seven tracks is a minimum is situated in longitude 146° 20.5′, latitude +17° 25.3′, Eq. 1875.0, and is shown by a circle having the size of its probable uncertainty. The longitude of the Sun for the mean of the seven times is 233° 9.4′, its radius vector logarithm 9.99505, and hence the mean orbit of the above seven meteors around the Sun, always assuming the period to be 33.25 years, is found to be:

Perihelion	1898, Nov. 10.354, M. T. Greenwich	
Long. of Perihelion	46°45′	Eq. 1875.0
Node	233 9	
Inclination	162 33	
Eccentricity	0.9046	
Period	33.25 years.	

The elements deduced by Oppolzer for Comet 1866 I are:

Perihelion	1867, Jan. 11.171, M. T. Berlin	
Long. of Perihelion	42°24.0′	Eq. 1866.0
Node	231 26.1	
Inclination	162 41.9	
Eccentricity	0.9054	
Period	33.176 years.	

According to Berberich, the principal perturbations of the meteors arriving on Nov. 15, 1898 are:

In Long. of Perihelion	+	110.1′
Node	+	50.2
Inclination	+	42.7
Eccentricity	+	0.0016

and the precession from 1866 to 1875 amounts to +7.3′, +7.4′, and +0.1′ for Perihelion, Node, and Inclination respectively, so

that we might expect changes in the direction actually found in general from Oppolzer's orbit.

Unfortunately only one of the Leonid meteors was secured at both stations, so that we can deduce but one height of apparition and disappearance. Trail III on the Observatory plates (the one shown on Plate I of Vol. IX) combined with IV gives for these two heights, 59.97 and 53.14 geog. miles, or 111.2 and 98.6 kilometers, a result in near accordance with that derived from the Perseids.

In this preliminary communication it seems hardly worth while to enter closely into the accuracy with which the radiant is derived from the photographs, but I may mention that, as can be inferred from the diagram A, the apparent radiant on Nov. 14 from trails II to VII, is located with a probable error of about $\pm 2'$ in each coördinate. The trail on Nov. 15, No. VIII, deviates considerably, but diagram B shows that this deviation is due mainly to the change of position of the Earth.

THE DIRECT CONCAVE GRATING SPECTROSCOPE.

THE ULTRA-VIOLET HYDROGEN SERIES.

THE GREAT NEBULA OF ORION.

By S. A. MITCHELL.

A NOTE has been published in this JOURNAL regarding the application of the concave grating to stellar photography.[1] The grating has been tried in several different ways, but the best results are obtained by using it as a direct grating spectroscope; that is, the light from the star falling directly on the grating, is diffracted and brought to a focus on the photographic plate.

This method was tried successfully at the Johns Hopkins University (*loc. cit.*). A small grating of one meter radius of curvature, with 15,000 lines to the inch, was employed. Although this grating had a ruled surface of only 1×2 inches, a photograph of Sirius was obtained which showed sixteen lines of the hydrogen series. This seemed so promising that a large grating was made, having a ruled surface of $2 \times 5\frac{3}{4}$ inches. This grating has a radius of curvature of one meter, and is ruled with 7219 lines to the inch. It was mounted on the 9½-inch Hastings refractor of the Johns Hopkins University. Experiments were continued, the results obtained being very good, considering the situation of the Observatory. The telescope is on the sixth floor of the Physical Laboratory, which is continually subject to the jars, the dust, and the glare of the city.

In November 1898 an opportunity was presented, through the kindness of Professor Hale, of working at the Yerkes Observatory. The spectroscope was mounted on the 12-inch Brashear refractor, and experiments have been continued through the past winter and spring.

The mounting for the grating is exceedingly simple,

[1] POOR and MITCHELL, "The Concave Grating for Stellar Photography," this JOURNAL, **8**, 157, 1898; *Monthly Notices*, March 1898.

consisting only of the ordinary grating holder with side and back adjustments, and a focusing apparatus for the plate-holder. By these means the spectrum is focused on the photographic plate, and the plate and grating made parallel, thus giving a "normal" spectrum (*loc. cit.*). The whole is inclosed in a wooden box clamped to the tube of the equatorial, which is used merely for the purpose of following the star.

The box is fastened to the tube in such a way that the ruled lines of the grating are parallel to the equator. The slight irregularities of the driving clock then do not alter the definition of the photograph. To insure good definition, it is necessary only to follow accurately in declination, and this is accomplished very easily by using the filar micrometer in connection with the equatorial. It is found[1] that for this grating the astigmatism amounts to 0.003 inches, and therefore, if the star were followed accurately, the resulting spectrum would be of this width. Consequently the spectrum was generally widened by regulating the driving clock so as to cause the star to trail in right ascension.

The above apparatus is extremely simple, and it is a remarkably easy undertaking to photograph the spectrum of a star.

Since the light from the star passes through neither objective nor prism trains, there is no absorption of the ultra-violet light, except the small selective absorption of speculum metal; and since no slit is used no light is cut off by this means. Consequently the photographic spectrum extends far into the ultra-violet. When we consider that the photograph is obtained in so simple a manner, and that the resulting spectrum is "normal," we perceive some of the advantages of the grating spectroscope.

With the large grating used, the spectra on both sides were about equally bright. The photographic extent of the first order was about 1¼ inches, that of the second order about 2½ inches, using Seed's Gilt-Edge plates. In the second order the distance from $H\beta$ to $H\gamma$ was about 0.6 inches; from $H\beta$ in the first order to $H\beta$ in the second the distance was 2.8 inches. The first and

[1] MITCHELL, "Theory of the Concave Grating," this JOURNAL, **8**, 110, 1898.

second orders together had an extent of about 4 inches, and consequently, using a photographic plate 1 × 5, both spectra could be photographed on the same plate. The plates were bent as closely as possible to the proper radius (this JOURNAL, March 1898).

By adjusting grating and plate-holder so as to make them parallel, the second order spectrum was found to be "normal." These adjustments were very easily made.

Photographs were made of a large number of stars of different magnitudes, the results of a few of the plates being as follows:

Plate	Date	Star	Exposure	Width of spectrum	Results
	1898		m.	in.	
108	March 31	Vega	20	0.02	14 Hydrogen lines, K
115	April 2	Vega	10	.01	14 Hydrogen lines, K
133	April 6	Capella	40	.02	F, G, h. H. K. 125 lines between F and H. (Night very hazy)
136	April 8	Spica	40	.02	12 Hydrogen lines, K; 10 other lines
166	Dec. 11	Sirius	2	.005	14 Hydrogen lines, K
	1899				
177	Jan. 11	Capella	5	.01	Fully exposed
190	Jan. 18	Rigel	5	.01	21 Hydrogen lines, K; many other lines. (Measured for ultra-violet hydrogen series)
191	Jan. 18	Sirius	5	.01	Excellent definition. 75 lines between $H\beta$ and $H\gamma$
214	Jan. 31	Sirius	15	.04	
214	Jan. 31	Rigel	15	.02	Excellent definition. 21 Hydrogen lines
222	Feb. 6	Sirius	15	.03	
224	Feb. 6	Procyon	25	.02	
227	Feb. 7	Orion nebula	200	—	Hydrogen series as far as $H\rho$. Temperature —11° F.
230	Feb. 11	Orion nebula	210	—	Temperature —18° F.
238	March 6	ϵ Orionis	65	.01	
239	March 6	Regulus	34	.01	13 Hydrogen lines, K
240	March 6	ϵ Ursae Majoris	66	.015	14 Hydrogen lines, K
241	March 6	Spica	41	.015	
242	March 6	Vega	13	.02	15 Hydrogen lines, K
248	March 23	α Cygni	20	.02	18 Hydrogen lines; many fine lines
252	March 23	ϵ Ursae Majoris	30	.01	
253	March 28	Procyon	25	.02	
254	March 28	Regulus	40	.02	
259	March 31	η Ursae Majoris	80	.02	
273	April 28	α Aquilae	40	.02	Night hazy
275	May 1	Arcturus	30	.01	

Most of the photographs were taken by allowing the star to trail slightly in right ascension, thus widening the star's spectrum. Account must be taken of this fact in considering the length of exposure.

These plates show, (1) the large extent of the ultra-violet light, (2) excellent definition, (3) that the exposure necessary compares very favorably with that in other forms of spectrographs. To show the excellence of the definition, it may be stated that on photographs of Sirius about 75 lines were measured between $H\beta$ and $H\gamma$. These lines were very sharp and narrow. (The results of these measures will be communicated at a later date.)

Twenty-one lines of the ultra-violet hydrogen series were measured—the series therefore is extended as far as $H\chi$. The series was seen to extend farther than this, the head of the series being measured at λ 3646.1, but the dispersion was not sufficient to separate the single lines. Of the different white stars photographed, the hydrogen lines in Rigel were found to be the sharpest and the best for measuring. The following measures are the results from three plates of Rigel.

Measuring lines of known wave-length in different spectra, it was found that the spectra were absolutely normal, and these measures are accordingly made on that assumption.

The hydrogen series is found from the well-known law of Balmer. Using Rowland's values of the first two lines of the series—Ha and $H\beta$, which are standards—the value of the head of the series is λ 3646.140. From the values of Ha, $H\beta$, and $H\gamma$, the value is λ 3646.138. From the first five lines of the series—Ha to $H\epsilon$—the value of the head of the series comes out as λ 3646.143. The series was calculated from the value $\lambda = 3646.140$, and the results are given in the following table.

The first column contains the designations of the hydrogen lines, the second the values of the wave-lengths as measured from plates of Rigel, the third the values obtained from Balmer's law, the fourth column gives the difference between the measured values and those derived from the law.

ULTRA-VIOLET HYDROGEN SERIES.

Line	Measured λ	λ from Balmer's law	O—C
$H\beta$	4861.5	4861.5	0.0
$H\gamma$	4340.8	4340.7	+0.1
$H\delta$	4102.1	4101.9	+0.2
$H\epsilon$	3970.5	3970.2	+0.3
$H\zeta$	3889.2	3889.2	0.0
$H\eta$	3835.4	3835.5	—0.1
$H\theta$	3798.2	3798.1	+0.1
$H\iota$	3770.8	3770.8	0.0
$H\kappa$	3750.2	3750.3	—0.1
$H\lambda$	3734.4	3734.5	—0.1
$H\mu$	3722.0	3722.1	—0.1
$H\nu$	3712.0	3712.1	—0.1
$H\xi$	3704.1	3704.1	0.0
$H\omicron$	3697.4	3697.3	+0.1
$H\pi$	3691.5	3691.7	—0.2
$H\rho$	3686.9	3687.0	—0.1
$H\sigma$	3682.9	3683.0	—0.1
$H\tau$	3679.4	3679.5	—0.1
$H\upsilon$	3676.4	3676.5	—0.1
$H\phi$	3673.9	3673.9	0.0
$H\chi$	3671.0	3671.6	—0.6
Head of series	3646.1	3646.1	0.0

The residuals given in the fourth column speak for themselves, and show the value of the measures made with the grating used in this manner.

Probably the most interesting photographs, however, were those of the Great Nebula of Orion. Two photographs were obtained, one on February 7, the other on February 11. These are especially interesting on account of the controversy which has been going on in regard to the intensities of the different lines of the spectrum. Although Professor Keeler has definitely settled this question by means of his admirable photographs with color screens, still a few words on the present photographs may not come amiss. Photographs with a slitless spectroscope—an objective prism or a grating—show a spectrum made up of images of the object photographed. Thus, in the present photographs, the lines of the spectrum consist of a series of images of the nebula. Although the stars in the immediate neighborhood of the nebula give spectra which overlap the spectrum of the

nebula itself, still it is easy enough to distinguish the bright images of the nebula from the spectra of the stars.

There are marked differences in the appearances of the different images making up the spectrum. The plates used were Seed's Gilt Edge. The brightest image in the whole spectrum is that due to the $H\gamma$ radiation. This is a trifle brighter than the chief nebular line λ 5007 and $H\beta$, which appear on these photographs as a whole about equally bright, and of about the same intensity as the violet image λ 3727. The hydrogen lines, from $H\delta$ on, appear as a series gradually decreasing in intensity. The series has been traced as far as $H\rho$. Although the images are small—the focal length of the grating is about 20 inches—they are sufficient to show large differences in the appearances of the different lines. The violet line λ 3727 has the greatest extent. The faint outlying regions show in this line a greater intensity and a greater extent than in $H\beta$. The Huyghenian regions appear about equally intense in λ 3727 and $H\beta$, and of slightly less intensity than the same regions in $H\gamma$. The extent of $H\gamma$ is about the same as that of $H\beta$.

Of the greatest interest are the comparisons between the $H\beta$ line and the chief nebular line, there being marked differences of intensity in different regions. While in the Huyghenian region the line λ 5007 is brighter than $H\beta$, in the faint outlying portions $H\beta$ is clearly brighter than λ 5007. Taking into account the rapid falling off of the sensitiveness of Seed's plates from $H\beta$, we come to conclusions which are perfectly in accord with those of Professors Campbell and Keeler.

Portions brighter than the rest of the region, which show in these photographs as a double bright line, appear in the different images. By means of these it was possible to measure the wave-lengths with the accuracy shown in the accompanying table. In fact, in the fainter images of the spectrum, this double bright line was about all that appeared.

In the following table, in column 1 is given the line with its estimated intensity on a rather arbitrary scale, where 100 denotes the brightest line and 1 denotes a line just visible; column 2 gives the wave-lengths measured from the plate taken

February 7; column 3, those measured from the plate taken February 11. For the sake of comparison, in column 4 is given the wave-length of the hydrogen lines, and of the lines measured by Keeler (K)[1] and by Campbell (C)[2].

Possibly other lines than these given appear in the spectra, but these were the only lines measured with certainty on both plates.

WAVE-LENGTHS OF LINES IN THE GREAT NEBULA OF ORION.

Line and Intensity		February 7	February 11	Keeler and Campbell
	Chief Nebular (80)	5007.0	5006.9	5007.0 (K)
	(20)	4959.0	4959.0	4959.0 (K)
Hβ	(80)	4861.5	4861.5	4861.5
	(2)	4713	4713	4716 (K); 4713 (C)
	(1)	4665	4666	466± (K); 4664 (C)
	(1)?	4566	4566	.....
	(3)	4471.9	4471.6	4471.2 (K); 4472 (C)
	(2)	4391	4390	4389 (C)
	(1)	4364	4364	4365 (K); 4364 (C)
Hγ	(100)	4340.8	4340.7	4340.7
	(1)	4144	4144	4144 (C)
	(1)	4123	.....	4121 (C)
Hδ	(20)	4101.6	4101.8	4101.8
	(1)	4067	4068	4069 (K); 4067 (C)
	(1)	4026	4026	4026 (K); 4026 (C)
He	(10)	3970.1	3970.8	3970.2
Hζ	(5)	3889.0	3889.1	3889.1
	(5)	3868.6	3868.4	3868.9 (K); 3869 (C)
Hη	(3)	3835.6	3835.4	3835.5
Hθ	(2)	3797.9	3798.1	3798.1
Hι	(2)	3771.2	3770.4	3770.8
Hκ	(2)	3750.3	3750.2	3750.2
Hλ	(2)	3734.5	3734.2	3734.2
	(80)	3727.5	3727.3	3726.5 (K); 3727 (C)
Hμ	(2)	.....	3722	3721.9
Hν	(1)	3712	3710	3711.8
Hξ	(1)	3706	3704	3704.0
Ho	(1)	3697	3698	3697.4
Hπ	(1)	3692	3691	3691.5
Hρ	(1)	3686	3687	3686.9

[1] *Astronomy and Astro-Physics*, **126**, 484, 1894.

[2] *Astronomy and Astro-Physics*, **125**, 392, 1894.

ABSOLUTE WAVE-LENGTHS.

The fundamental formula for the determination of wave-lengths (this JOURNAL **8,** 110, 1898), is:

$$\lambda = \frac{\omega}{N}(\sin \mu + \sin \gamma)$$

where λ is the wave-length, ω the grating space, N the order of the spectrum, μ and γ the angles which the line λ and the source of light respectively make with the axis of the grating, which is the axis of reference.

Absolute wave-lengths are obtained by the use of the grating in two ways, according as we make (1) $\mu = 0$, or (2) $\gamma = 0$. If we make $\mu = 0$, the line λ is then on the axis of the grating, or in other words grating and camera are parallel. The grating has been used in this manner in the above work. For variations of μ for different lines in the spectrum, such that cos μ does not exceed unity, it is found that the spectrum is "normal" (this JOURNAL, **7,** 157, 1898). As noted above, the whole second spectrum was found to be "normal." Since we do not know for which line λ the angle $\mu = 0$, we have to find our absolute wave-lengths by taking advantage of the fact that the second order spectrum is "normal." This is done in the following manner:

As above, the first and second orders of spectra are photographed on the same plate. Take any two lines in both spectra for which we wish to know the absolute wave-lengths. Measurements on these lines will give us three quantities, viz., the distance from the first to the second order for the two lines respectively, and the distance between the two lines in the second order. From these three quantities the absolute wave-lengths may be obtained. The formulae necessary are easily derived, and the necessary calculations to obtain absolute wave-lengths are short. Theoretically this might seem to be a valuable way in finding motion in the line of sight. Practically, however, it is not. The method is a differential one, and the

small errors in setting made in the measurements are sufficient to spoil the theoretical beauty of this method.

The second method of determining absolute wave-lengths is not a differential method but a direct one. If we make $\gamma = 0$, the source of light is on the axis of the grating, and our fundamental formula reduces to:

$$\lambda = \frac{\omega}{N} \sin \mu.$$

To obtain absolute wave-lengths by this very simple formula, it is necessary only to adjust the apparatus so that the source of light is on the axis of the grating, or what comes to the same thing, the image of this source, formed at the focus, is on the axis of the grating. This adjustment is accomplished very easily by holding a candle at the radius of curvature of the grating and turning the grating until the candle and its image coincide. (This is the same adjustment as in Rowland's ordinary form of grating mounting.) If we put the center of the photographic plate on a line between the candle and the center of the grating, and make the plate parallel to the camera, the apparatus is in perfect adjustment.

This method was tried. The light from the star after just grazing the camera — so as not to cut off any light from the grating — falls on the grating. The grating is slightly tilted by the back adjustment screw. The light is brought to a focus on the photographic plate. At the center of the plate we will have the image of the star formed by direct reflection at the grating, and equidistant on either side the first order spectra of the star.

In the above formula, to find the angle μ for any line λ, we have simply to measure the distance of this line from the image of the star formed on the plate, which distance will give us directly the angle μ, since we know the focal length of the grating. If, instead of measuring the distance of the line λ from the central image, we measure it from the line λ in the other spectrum, we consequently measure double the distance, thus finding the angle μ, and so increase the accuracy of our measurements. Thus this method of finding absolute wave-lengths with

the determination of motion in the line of sight is a very simple one. The grating used in this manner does not give a normal spectrum.

Owing to the fact that the dispersion in the first order with the grating used was not sufficient for the accurate determination of motion in the line of sight, this method was not tried further than to show that the definition obtained is excellent, and that the central image is small enough to be measurable if necessary. Using a grating of larger radius of curvature we would have the double advantage of increasing the dispersion and also of being able to bend our plates more nearly to the proper radius. There would be the disadvantage of lengthening the exposure, but this will be overcome when larger gratings are ruled.

The above results were obtained by the use of a grating of $2 \times 5\frac{3}{4}$ inches, or a light gathering surface of $11\frac{1}{2}$ inches, which is less than the surface of the objective of a 4-inch telescope. Taking into account the exposure required as well as the excellent definition obtained and the accuracy of the measurements, we see the value of this form of spectroscope.

The advantages of the direct grating spectroscope may be summed up as follows :

1. Simplicity of the apparatus.
2. The ease with which a star can be accurately followed.
3. The spectrum is readily widened by regulating the driving-clock of the equatorial.
4. Great extent of the ultra-violet light.
5. Excellent definition.
6. The necessary exposure compares very favorably with that of other forms of spectroscopes.
7. The spectrum is normal.
8. Absolute wave-lengths, and consequently the motion in the line of sight, can be determined.

With a grating of say 10×15 inches, considering the exposure, the accuracy of the wave-lengths coming from good definition and a normal spectrum, and also the ease and accuracy

with which we could determine motion in the line of sight, the general results would probably be comparable in excellence with the results obtained by any spectroscope at present in use, including those attached to the Lick and Yerkes telescopes.

THE UNIVERSITY OF CHICAGO,
YERKES OBSERVATORY,
June 1899.

ON THE DISTRIBUTION OF THE ENERGY IN THE SPECTRUM OF THE BLACK BODY AT LOW TEMPERATURES.[1]

By F. PASCHEN.

MY observations on the energy spectra of different solid bodies[2] make it seem possible that the law derived by W. Wien[3] represents the emission of "the absolutely black body." In Wien's formula,

$$J = c_1 \lambda^{-5} e^{-\frac{c_2}{\lambda T}}, \qquad \text{(I)}$$

where $J d\lambda$ is the energy between wave-lengths λ and $\lambda + d\lambda$ at the absolute temperature T, and c_1 and c_2 are constants, if we substitute λ^a for λ^{-5}, the value of a changing from body to body, my former observations are represented by the formulae. The value of a decreased from 6.4 to 5.2 in passing from reflecting platinum to strongly absorbing carbon.

The deviation of my former observations from the theoretical laws was such that that which was theoretically well-founded was not confirmed with certainty, while on the other hand that which was uncertain theoretically was rendered probable by the observations. The comprehensive formula of the law of emission is not supplied by theory in an unquestionable manner. If we assume the validity of the formula with the constant a derived from the experiments, then a must have the value 5, for Wien has proven that the intensity J_m of the maximum of energy varies as the fifth power of the temperature, which is the case in the assumed formula only when $a = 5$. In addition to this relation

[1] *Sitzungsberichte der Berliner Akademie;* Session of the physical-mathematical section on April 27, 1899.

[2] *Wied. Ann.*, **58**, 455, 1896; **60**, 662, 1897. (The latter will be referred to as *loc. cit.*)

[3] *Wied. Ann.*, **58**, 662, 1896.

$$\frac{J_m}{T^5} = \text{constant}, \qquad (1)$$

two further relations are firmly established by Wien, or follow from his accurate derivations, viz.:

$$\lambda_m \times T = \text{constant}, \qquad (2)$$

or the wave-length of the maximum of the energy curve for temperature T is inversely proportional to the temperature; and (3) the relation that the ratio of the intensity J of wave-length λ to the intensity J_m of the energy maximum of wave-length λ_m, or $\frac{J}{J_m}$, is a function of $\frac{\lambda}{\lambda_m}$, and for the energy curves at different temperatures always the same function of $\frac{\lambda}{\lambda_m}$. In a logarithmic representation all the energy curves must be congruent.

I found laws (2) and (3) to be confirmed. The experiments gave for the last relation (3) the function

$$\frac{J}{J_m} = \left\{ \frac{\lambda_m}{\lambda} e^{\frac{\lambda - \lambda_m}{\lambda}} \right\}^a, \qquad (4)$$

which theory left indeterminate. It was only on making certain doubtful assumptions that Wien succeeded in deducing formula I, and we may regard formula (4) with a set equal to 5 as the result of Wien's research.

We therefore have to investigate whether this function (4) holds good with the value of $a = 5$, if we approach closely to the radiation of the absolutely black body. This seemed probable from my experiments, since the function was valid for radiating bodies of very different absorptive power, and since a had already taken the value 5 for the blackest body. The confirmation of relation (1), which according to theory must hold first of all, will be a crucial test whether the arrangements of the experiments sufficiently comply with the postulates of the theory; for the constant c_2 of formula I can be determined from the measures with sufficient accuracy only when this relation — in my experience the most difficult to realize — also holds good.

In attempting to answer these questions I first determined the energy spectra of cavities, the sides of which were heated by

baths. These experiments gave a somewhat closer approximation to law I than my former observations, as the relations 2, 3, and 4 appeared to be quite fully satisfied with $a = 5$, and as the observations were best represented by my previous formula with a value of about 5.2 for the exponent of λ; but it was impossible, however, to find the intensity of the maximum of energy exactly proportional to the fifth power of the temperature without overstepping the limits of error very considerably. The discrepancy might be due to the imperfect realization of the radiation of the black body, as the cavities had openings. When these were very much reduced, however, the emergent radiation still gave the discrepant result as before. The only thing in the experimental arrangements that could be held responsible for this was a variability of the absorptive power of the exposed bolometer strip with the wave-length. It therefore remained either to determine this or to so arrange the receiver of the radiation that it should constantly absorb the incident radiation as nearly completely as possible. I adopted the latter method and beg to show in what follows how far I have been able to approach to my aim. The experiments I communicate refer to the region of low temperatures and long wave-lengths. These experiments are the best adapted for judging of the blackening of the receiver, since my ordinary bolometers deviate most from my blackest ones just at this region.

ARRANGEMENT OF THE EXPERIMENTS.

The spectroscopic apparatus employed includes a fluor-spar prism loaned me by the firm of Carl Zeiss, which I had used before, and two silver concave mirrors of 35 cm focus, with precisely spherical figure, and so arranged that the astigmatism of the image was reduced as much as possible. The spectrum thus produced was so exceedingly sharp that the two broad absorption bands of aqueous vapor in the air of the room at $\lambda 6.0\,\mu$ and $6.5\,\mu$ were resolved into numerous sharply defined bands, while between them at $\lambda 6.26\,\mu$ was a place without appreciable absorption. The absorption band of carbon dioxide of the air

of the room appeared narrow like a line, but at its deepest part extinguished more than two thirds of the original energy.

All of the bolometers with which the results below were obtained were simply strips of platinum of $\frac{1}{2000}$mm thickness and most had a breadth of 0.5mm, corresponding to an angle of 5 minutes in the spectrum. The piece upon which radiation fell had as accurate a rectangular form as possible. The slit-width was altered until the energy curve of a line (or of the image of the slit when the prism was removed) became as nearly as possible an isosceles triangle, as the correction for the impurity of the spectrum due to the width of the slit can be simply calculated for this case.[1] The exposed surface of the bolometer was blackened either with lampblack or according to the Lummer-Kurlbaum[2] method with platinum black. The layer of black was given two or three times the thickness prescribed in their rule or employed in my earlier bolometers. The extremely slight thickness of the bolometer strip gives the advantage that in spite of the thick covering with black the galvanometer deflection, even with a period of six seconds, behaves just as on breaking a shunt across the branch of the bolometer if the conductors to the sensitive parts are screened by metallic diaphragms, and protected above and below from the latter for a space of some 0.5 mm.

To produce a further effect of blackening, the bolometer strip was placed, according to the principle proposed by myself (*loc. cit.* p. 722) with its middle exactly at the center of a reflecting hollow shell which had a small aperture for the admission of the radiation. Only that hemisphere of the shell was present on which the radiation reflected from the strip could fall, while the strip was so fixed that it could reflect toward all possible parts of the hemisphere. The frame of the bolometer could be moved by a micrometer until the strip covered its image. For reflecting hemispheres I used one of 45 mm diameter with poor polish and an inaccurate surface. A second one, cut exactly spherical by

[1] C. RUNGE, *Schlömilch's Zeitschrift für Math. und Phys.*, **42**, 205, 1897.

[2] F. KURLBAUM, *Proceedings of the Physical Society of Berlin*, p. 11. June 14, 1895.

Zeiss of Jena, of diameter 50 mm, had a splendid polish.[1] Both were of German silver. For the present research with the bolometer strips from 5 to 7 mm long and comparatively broad, it was sufficient to project the spectrum sharp in the plane of the strip. If one is to work with higher dispersion it is better to throw a sharp image of the spectrum in the plane of the slit of the hemisphere, and to make this slit equal to the image of a line. A very perfect arrangement in respect to the blackening of the strip and the projection of the spectrum is obtained if a bolometer is placed in the central plane of the hemisphere, of such width that the incident radiation does not fully cover its strip in length and breadth, and so that the radiation diffusely reflected from the strip always returns upon sensitive parts in spite of the aberrations for rays out of the center of the sphere. This also has the advantage of allowing, even with high dispersions, the use of a sensitive surface bolometer, which of course must be opaque for its whole breadth. The visual observation of the spectrum projected upon the exterior wall of the hemisphere is effected by a properly adjusted mirror. Since the bolometer strip reflected upon itself is heated by the current, and gets a great part of its radiation back again, the equality of the resistances of the bolometer will be disturbed. In order that the second strip should also get back at least a part of its radiation, I attached a plane silver mirror parallel to the central plane at the position of its image. With the strongest currents permissible, however (0.05 amperes), no such inequality of resistances arose to render the bolometer useless, so that the mirror was unnecessary.

I used for galvanometer a newly constructed instrument which is more sensitive than my former one,[2] and is rendered so far astatic that its directing force is chiefly due to the quartz fiber. Only in this way was it possible to work with so delicate a galvanometer at a place strongly affected by the earth-currents from a near by electric street railway.

[1] Viewed with a microscope magnifying about 150 times, the image of a fine thread in the center appeared as sharp as the thread itself.

[2] *Wied. Ann.*, **50**, 417, 1893.

The radiating cavities were cylindric or pear shaped and of such size that the distance was from 10 to 13 cm from the rear wall to the aperture of about 1 sq. cm area. Only a part of the rear wall could send radiations through the suitably diaphragmed aperture. A larger vessel enclosed the hollow radiator, and served either to boil a liquid or to contain the vapors led in from a special vessel in which the liquid was boiled. The hollow radiator was always in the vapors. The arrangement for different temperatures was briefly as follows:

1. 100° C. The steam from water boiling in a flask was led into the enveloping vessel. Hollow radiators of copper oxide, lampblack and platinum black were investigated.

2. 190° C. Impure commercial aniline was boiled with a return condenser in the metallic enveloping vessel. The temperature of the cavity was determined each time by a thermometer certified by the *Reichsanstalt.*

3. 304° C. Arrangement the same as last, with impure commercial di-phenylamin. Cavities covered with copper oxide or lampblack.

4. 450° C. Sulphur was boiled in the jacket of a double-walled glass vessel with a condensing tube. The inner vessel constituted the cavity. Its surface, which had been roughened by etching, was covered by a thick layer of copper oxide. In several experiments a lampblack layer was put on over this, but it disappeared very quickly.

The glass vessel was closely surrounded by the metallic jacket. The flame played freely only under the boiling sulphur. With this arrangement the temperature of the interior of the cavity was from 449° to 451° C., according to the atmospheric pressure and the arrangement of the flame, being indicated by a mercury thermometer extending to 550° C. (certified by the *Reichsanstalt*) or by a thermo-element referred to this.

Differences of temperature as small as 1° C. with aniline, and 3° C. with sulphur, were determined by a thermo-element in the interior of the cavity, in some cases. The temperature was then taken at the point of the rear wall opposite the aperture. All of

the cavities could easily be provided with smaller apertures by the introduction of diaphragms. Some of the cavities in zinc were covered with platinum black by the reduction of a solution of 1 per cent. platinum chloride and 0.1 per cent. lead acetate on the zinc wall. The blackening thus obtained almost surpasses that of the electrolytic process of Lummer and Kurlbaum if the basic salts of zinc have been removed with dilute acetic acid. The cavities were placed in front of the spectroscope in such a way that the radiation from the aperture entirely filled a fixed diaphragm before the prism.

Energy curves only were observed, the sensitiveness of the bolometer in the different series being compared by the well-known shunt method of Ångström, or by comparative observations of a constant source of radiation, or by the measurement of the bolometer current. I proceeded along the spectrum to $\lambda\, 5\,\mu$ by steps of the width of the bolometer strip (an angle of about 5′), since the correction for impurity of the spectrum can then be very easily and accurately computed, as shown by Runge.

The first of the strong absorptions of aqueous vapor begins at $5\,\mu$[1]. At $6.26\,\mu$ there is a narrow place between the two bands which shows no absorption in my spectrum, but at which the correction for impurity of spectrum reaches a considerable amount on account of the falling off of energy on the two sides. Beyond the second strong water absorption, a region begins at about $7.7\,\mu$, which exhibits as far as about $9.3\,\mu$ no appreciable absorption by the air of the room, but does show absorption by the substance of the prism. In this region the four wave-lengths, $7.738\,\mu$, $8.246\,\mu$, $8.806\,\mu$ and $9.324\,\mu$ were investigated, account being taken of the absorption by the prism.

For the calculation of the normal energy spectrum, in which the energy contained in a constant, narrow range of wave-lengths is considered a function of the wave-length, I made use of my new determination of the dispersion of fluor-spar, in which the

[1] PASCHEN, *Wied. Ann.*, **53**, 335, 1894. Graphical representations of the energy curve covering these regions of absorption may be found in *Wied. Ann.*, **52**, Plate, Fig. 1; **51**, Plate, Fig. 2, Curve 1.

prism and grating spectra had considerably greater dispersion and sharpness than in my earlier determinations, so that the results had errors from two to three times smaller than before. My first measure[1] proved the more correct, and the accuracy of the determinations of the constants of Ketteler's dispersion formula was increased, so that the differential quotients of this formula which were used in the reduction have throughout the spectrum errors less than those of observation (at least in this research).

In addition to the corrections I have previously dealt with in detail, for width of the slit, for reflection from the surfaces of the prisms and mirror, and for the spectrum of the slide[2] in front of the slit serving as a zero point, the absorption of the prism was eliminated for the region from 7.7μ to 10μ (see below).

MEASUREMENTS.

I will cite only the result of one of the series of measures with an ordinary lampblacked bolometer, for which series the radiating cavities were arranged in the same way as for the later experiments. Other measurements with ordinary bolometers and cavities heated by baths gave similar results.

BOLOMETER STRIP WITH A THICK LAYER OF LAMPBLACK OF AN ANGULAR WIDTH IN THE SPECTRUM OF FIVE MINUTES.

Temperature	λ_m	$\lambda_m \times T$	J_m	$\frac{J_m}{T^5} \times 10^{14}$
304.0° C. = 577.0° Abs.	4.818	2780	1.546	2.421
190.5 463.5	5.964	2764	0.4083	2.105
99.9 372.9	7.393	2757	0.136	2.08

Radiating cavity of copper oxide.

The theoretical curve of formula (4) with $a = 5$ gives the above results from the observations. The product of $\lambda_m \times T$ is

[1] *Wied. Ann.*, **53**. 301, 812, 1894.

[2] When this was let down an almost entirely closed cavity, containing a thermometer, presented itself to the slit. The intensity of its spectrum was computed for all the temperatures read and for all the wave-lengths, according to the principles of this research, and was then added to the observed intensity. The procedure is similar in testing the law of total radiation.

accordingly constant within the error of determination of λ_m,[1] but the intensity J_m of the wave-length λ_m of the maximum of energy, measured in arbitrary units, is not proportional to T^5 within the limits of error. Hence no certain conclusion can be drawn from these and similar measures.

BOLOMETER STRIP I, COVERED WITH A THICK LAYER OF LAMPBLACK, 5.0′ BROAD, AT THE CENTER OF THE POORLY REFLECTING HEMISPHERE.

Temperature		λ_m	$\lambda_m \times T$	J_m	$\frac{J_m}{T^5} \times 10^{14}$	
450.0° C. = 723.0° Abs.		4.009	2898	4.798	2.430	Lampblack in cavity[2]
. . .	. . .	. . .	. . .	. . .	. . .	
304.1	577.1	5.010	2891	1.547	2.416	Copper oxide in cavity
304.1	577.1	5.010	2891	1.544	2.402	
304.1	577.1	5.018	2896	1.567	2.438	
304.1	577.1	5.013	2893	1.553	2.419	Mean
. . .	. . .	. . .	. . .	. . .	. . .	
191.0	464.0	6.224	2888	0.5206	2.421	Copper oxide in cavity
191.0	464.0	6.216	2884	0.5260	2.446	
191.5	464.5	6.197	2879	0.5296	2.449	
191.2	464.2	6.212	2884	0.5254	2.439	Mean
. . .	. . .	. . .	. . .	. . .	. . .	
99.7	372.7	7.727	2880	0.176	2.45	Copper oxide in cavity
99.8	372.8	7.732	2883	0.173	2.40	
99.7	372.7	7.736	2883	0.175	2.43	
99.7	372.7	7.732	2882	0.175	2.43	Mean

The results of these measures show a distinctly better accord with the theoretical laws. Since the value of the product $\lambda_m \times T$ has been very greatly increased by the blackening of the bolometer, one cannot tell whether the arrangement already so far satisfies the theoretical postulates that this new value can be considered as valid within the errors of observation. This

[1] These errors are larger than in the later measures, as the course of the curve was not as accurately reproduced by formula (4) as with the black bolometers.

[2] In two further series, at 450° C., the layer of copper oxide in the cavity was too thin, so that it was somewhat transparent at short wave-lengths; and it was only with long waves, which glass emits strongly, that it was sufficiently black, so that I obtained larger values for $\lambda_m \times T$ (up to 2928).

value seems, moreover, to be slightly variable with the temperature. Further objections are discussed later. Therefore further measures were made in which the blackening of both bolometer and cavity was more perfect.

BOLOMETER STRIP II, 5.0′ WIDE, COVERED THICKLY WITH PLATINUM BLACK, IN THE POORER REFLECTING HEMISPHERE.

Temperature		λ_m	$\lambda_m \times T$	J_m	$\frac{J_m}{T^5} \times 10^{14}$	
450.0° C. =	723.0° Abs.	4.021	2907	3.707	1.877	Thick layer of copper oxide in cavity
450.0	723.0	4.018	2905	——	——	Same, but smaller aperture, covered with lampblack
450.6	723.6	4.007	2899	3.741	1.886	
	Mean - -		2904			
304.0	577.0	5.012	2892	1.200	1.869	Lampblack in cavity
191.0	464.0	6.223	2887	0.403	1.873	
[191.0	464.0	6.310	(2927)	0.393	1.826]	Glass cavity, with walls etched rough and not blackened [1]
Mean of all - -			2894 [2]			

This bolometer could not bear the strength of current necessary for investigating the temperature of 100° C.

BOLOMETER STRIP III, 6.3′ WIDE, BLACKENED WITH PLATINUM, IN THE PERFECT HEMISPHERE.

Temperature		λ_m	$\lambda_m \times T$	J_m	$\frac{J_m}{T^5} \times 10^{14}$	
449.5° C. =	722.5° Abs.	4.019	2903	7.492	3.807	Copper oxide in cavity
304.1	577.1	5.009	2891	2.446	3.805	
190.7	463.7	6.227	2887	0.806	3.76	Platinum black in cavity
190.2	463.2	6.240	2891	0.806	3.78	
100.5	373.5	7.722	2884	0.279	3.83	
Mean - - -			2892			

[1] At large wave-lengths glass emits as strongly as lampblack. Since it is only in a slight degree transparent at 4.5μ, a glass cavity cannot be used even at 100° C. One gets too great a wave-length for the maximum of energy, and at short wave-lengths the energy curve falls off too abruptly, in so far as the bands of the spectrum of the heating vapors do not shimmer through.

[2] The mean at 450° C. was calculated once, the value 2927 not at all.

BOLOMETER STRIP IV, 5.0′ WIDE, BLACKENED WITH PLATINUM, IN THE PERFECT HEMISPHERE.

Temperature		λ_m	$\lambda_m \times T$	J_m	$\frac{J_m}{T^5} \times 10^{14}$	
450.5° C. = 723.5° Abs. (T)		3.998	2891	3.980	2.006	Lampblack in cavity
304.1	577.1	5.006	2889	1.219	2.012	
190.0	463.0	6.230	2885	0.422	1.984	Platinum black in cavity
190.7	463.7	6.230	2890	0.434	2.023	
99.8	372.8	7.742	2886	0.143	1.98	
Mean - - -			2888			

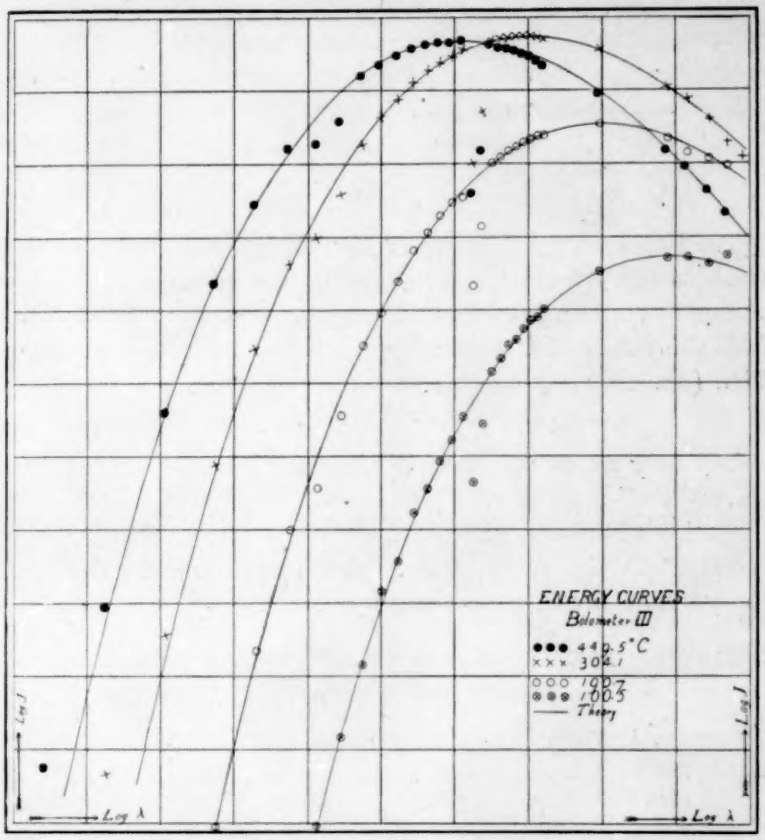
BOLOMETER STRIP V, 5.0′ WIDE, FIRST COVERED THICK WITH LAMPBLACK AND THEN WITH AN ELECTROLYTIC DEPOSIT OF PLATINUM BLACK, IN THE PERFECT HEMISPHERE.

Temperature		λ_m	$\lambda_m \times T$	J_m	$\frac{J_m}{T^5} \times 10^{14}$	
450.0° C. = 723.0° Abs. (T)		3.997	2890	4.207	2.121	Lampblack in cavity
304.1	577.1	5.012	2893	1.361	2.117	
189.4	462.4	6.249	2889	0.449	2.123	Platinum black in cavity
100.0	373.0	7.745	2889	0.153	2.12	
Mean - - -			2890	- -	2.120	

For judging of the accuracy of these results of the computations I would refer to the methods of reduction for the evaluation of the observed energy curves which are fully treated in my two previous papers. Here as there the most liberal use was made of the principle of the congruence of the energy curves represented logarithmically, which has been proven both theoretically and experimentally (Wien's third relation). Since formula (4) with $a = 5.00$ always reproduced the curve within the limits of error, excepting two or three unreliable extreme points at small wave-lengths, the value of λ_m and J_m could be determined with very great accuracy with a uniform regard of all the observed points. The figure gives the logarithmic representation, with $\log J +$ const. as ordinate and $\log \lambda$ as abscissa, of four energy curves obtained with bolometer III. The full line represents the curve of the formula (4) with $a = 5.000$, as it most closely fits the observations. The congruence is seen at once from the fact that this line is the same for all four curves.[1]

[1] In *loc. cit.* the energy curves were drawn one above the other in order to prove this.

Only the extreme points of short wave-lengths, where the measured energy at most amounts to $\frac{1}{100}$ of the maximum energy, deviate in the direction to cause a suspicion of diffuse light. This is entirely possible, since the maximum of energy of the observed prism-spectrum lies very near these points.



In order to exhibit the validity of law I in still another way, I have put together as "isochromatic lines" the intensities observed for the same wave-lengths from the four energy curves of the last series of measures (bolometer V), by treating $\log J$ as a function of $\frac{1}{T}$ for each wave-length. This was possible

for the reason that observed points of these series lay nearly at the same wave-lengths. An interpolation in the course of the curve would have been less convincing. Within the errors of measurement the isochromatic lines thus obtained were straight lines, and the values of c_1 and c_2 of formula I calculated from them agree, as was to be expected, with the more precise values of these constants given by the evaluation of the observations as energy curves, or hence given by the above tables.

The latter values are:

$$c_2 = \lambda_m \times T \times 5 = 2890 \times 5 = 14450$$

$$c_1 = \frac{J^m}{T^5}(\lambda^m \times T)^5 \times e^5 = 2.120 \times 10^{-14} \times 2890^5 \times 2.7183^5 = 634100.$$

The following table contains the summary of the observed values of log J included for the isochromatic lines. The calculation of the constants of the straight line for each wave-length yields the values of c_1 and c_2 given below them, the means of which (omitting the line at 1.887μ) are

$$c_2 = 14450, \qquad c_1 = 629100.$$

Since only a small number of observed points were included here, while all the observed points were taken into account in the calculation of the energy curves, we could not expect a better agreement in the value of c_1. The values derived from the energy curves are of course more accurate.

SUMMARY OF CERTAIN OBSERVED POINTS LOG J FOR ISOCHROMATIC LINES (BOLOMETER V).

T (Abs.)	$\frac{1}{T}$	λ=7.783	6.263	4.663	3.355	2.280	1.887μ
	0.00						
373.0	2681	0.184–1	0.132–1	0.846–2	0.155–2	—	—
462.4	2162	0.599–1	0.647–1	0.548–1	0.1305–1	0.043–2	
577.1	1733	0.951–1	0.0826	0.1240	0.9311–1	0.236–1	0.669–2
723.0	1383	0.2407	0.4375	0.5954	0.5775	0.1869	0.825–1
	c_2=	14440	14460	14450	14450	14450	14360 [1]
	c_1=	629400	636900	630300	629600	613700	738800 [1]

[1] These points at the extreme end of the rising branch are also here exhibited as lying too high (c_1 too large), and due to light of longer wave-length (c_2 too small.)

There is still one last consequence of the observations to be adduced in forming a judgment of the results obtained with the different arrangements of bolometers. I could not know on general principles whether a disturbing absorption of the air of the room would be present in addition to the absorption of the prism, in the region from 7.7μ to 10μ. As this subject has not been sufficiently investigated, since it was desired to include this region for the temperature 100° C., I have determined the factors for curves of higher temperature which were safely located by observations at short wave-lengths, with which factors the observed intensities beyond 7.7μ are to be multiplied in order that they should fit the theoretical curves. The factors thus determined therefore correct everything that would tend to make the observed intensities too small. I have on the other hand calculated by the law of absorption the factor which alone eliminates the absorption of the prism. It appeared that the factors for the blackest bolometers determined solely from the depression of the energy curve agreed thoroughly with the factors which corrected the absorption of the prism, so that it is improbable that there is any further absorption of any extent at the wave-lengths here observed. Lampblacked bolometer I in the poor hemisphere demands on the other hand still larger factors, which deviate increasingly with increasing wave-lengths, thus rendering necessary the conclusion that in this region this bolometer strip has less absorptive power than the others. The two following tables give information as to this:

I. DETERMINATION OF THE FACTORS FROM THE DEPRESSION OF THE ENERGY CURVE.

$\lambda =$	7.738	8.245	8.807	9.324
Bolometer I[1]	1.275	1.645	2.695	4.968
II	1.213	1.536	2.420	4.288
III	1.202	1.567	2.353	4.181
IV	1.205	1.563	2.388	4.436
V	1.217	1.555	2.449	4.454
Mean of II to V	1.209	1.555	2.403	4.340

[1] The energy curves of this bolometer are corrected with these factors.

2. CALCULATION OF THE FACTORS FROM THE ABSORPTION OF THE PRISM.

If the layer of fluor-spar of thickness l absorbs the fraction a, the emergent radiation from a prism of base B must be multiplied by the factor

$$\frac{\log (1-a)^{\frac{B}{l}}}{\left\{(1-a)^{\frac{B}{l}}-1\right\}\log e}$$

to obtain the incident radiation, as may be seen from a simple integration based on the law of absorption. The prism had its corners chipped, and had by no means a mathematically simple shape[1]. Measurement of the base gave in the mean a value of 43.0 mm. In the following table δ is the minimum deviation of the prism, λ the corresponding wave-length, and a the amount of the absorption I observed in a clear plate of fluorite of 4.056 mm thickness. The factor computed is calculated from the absorption, the factor observed is that determined from the energy curves above.

δ	λ	a	Factor comp.	Factor obs.
25°19.1′	7.738μ	0.0365	1.211	1.209
24 29.1	8.245	0.0885	1.570	1.555
23 29.1	8.807	0.1805	2.404	2.403
22 29.1	9.324	0.3125	4.042	4.340

Not until $\lambda = 9.324\mu$ does the observed intensity have to be increased more than corresponds to the absorption of the prisms. This may be due to the absorption of the air of the room or to an erroneous determination of the absorption of the fluor-spar, but it has no bearing on our conclusions. The energy curves obtained with an ordinary lampblacked bolometer must lie too low even at short wave-lengths, representing a power of absorption of the lampblacked surface decreasing for increasing wave-lengths. The layer of lampblack (through which the radiation has to pass twice) is therefore still considerably transparent even

[1] In *Wied. Ann.*, **53**, 333, 1894, I give the observations of the total loss of light, from which 5.05 per cent. is to be deducted for loss by reflection.

in a thick stratum, or else it has a rising reflecting power with increasing wave-length.[1] From several experiments the latter seems to me to be the case. Any such deviation at these long wave-lengths would be naturally expected, since the absorption of the lampblack bolometer strip in the reflecting shell can only be approximately equal to unity. Although the arrangement of bolometers II to V was different and represented an increase of the blackening (passing from the imperfect to the perfect hemisphere), nevertheless they absorbed almost equally in this spectral region, so that they seem to justify the conclusion that they all approximate "the absolutely black body" in this region so nearly that the casual deviation cannot produce appreciable error. I therefore believe that it is permissible to employ the results of the measurements with these bolometers in the determination of the constants of law I.

The constant c_2 is most accurately determined from these experiments by the fivefold value of the constant product $\lambda_m \times T$. This is in the mean:

For bolometer	II	III	IV	V	Mean of all.
	2894	2892	2888	2890	2891

The single values of a series of observations which deviate most widely from the mean of all are:

2907 (Bolometer II, 450°, cavity with layer of copper oxide);
2884 (Bolometer III, 100.5°, cavity with layer of platinum-black).

The first value is not entirely reliable, since it easily happened that some parts of the layer of copper oxide fell off, leaving the glass uncovered. This always gives occasion for an increase of the wave-length of the maximum.

I estimate the highest possible error of the mean of all as something like 3 per thousand.

Accordingly c_2 will be $= 5 \times 2891 = 14455$, with an error which at most I estimate at 40.

Since in formula I, $\frac{c_2}{\lambda T}$ represents a number, c_2 must have the

[1] K. Ångström has observed in thin layers an increasing transparency of lampblack with increasing wave-length. *Wied. Ann.*, 36, 720, 1889.

dimension: Wave-length × Temperature. Therefore

$$c_2 = 14455 \times \text{Degree Centigrade of absolute scale.}$$

The value of c_2 found by H. Wanner and myself[1] in a different region of temperature and wave-length by measurements of an entirely different kind was 14440, but it had considerably less accuracy.

The bolometric measurements at high temperatures recently published by O. Lummer and E. Pringsheim[2] yielded values of $\lambda_m \times T$ from 2837 to 2928, mean 2879, for one series, and for another series, in which the exponent 5.2 held good in the general formula I, five values between 2766 and 2986, mean 2876. From these they deduce for values of c_2, 14395 from the first series, and 14955 from the second series. Taking into account the errors possible according to the results of the observations and to the methods of reducing them, the agreement with my value seems to be good.

F. Kurlbaum[3] has measured the difference of the total radiation of a cavity coated with lampblack at 100° C. and at 0° in Watts × cm^{-2}, and has calculated that the heat equivalent of the total radiation emitted by a black body at the absolute temperature T amounts to

$$1.277 \times 10^{-12}\, T^4 \frac{\text{Gr. Cal.}}{\text{cm}^2 \times \text{sec.}}$$

or

$$5.32 \times 10^{-12} \times T^4 \text{ Watts} \times \text{cm}^{-2}.$$

By integrating formula I we get[4]

$$\int_0^\infty J\, d\lambda = 6\, \frac{c_1}{c_2^4}\, T^4 \qquad \text{(II)}$$

The constant c_1 has by formula I or formula II the dimensions

$$\text{Energy of radiation} \times (\text{Wave-length})^4.$$

Their numerical result depends, therefore, on the unit of the

[1] This JOURNAL, **9**, 304, 1899.

[2] *Verhandlungen der Deutschen Physikalischen Gesellschaft*, February 3, 1899.

[3] *Wied. Ann.*, **65**, 746, 1898.

[4] *Loc. cit.*, p. 666, where for $\alpha = 5$, $\Pi(\alpha - 2) = 6$.

energy of radiation. In my experiments this was arbitrary and was different for the six different bolometers, so that a more precisely definite value of the quantity c_1 could not be computed from these experiments alone. In order to refer the measurements to a definite unit of radiation let us now determine the value of c_1, with which the total radiation of my source of radiation reaches the quantity determined by Kurlbaum, so that the energy of radiation is measured in

$$\frac{\text{Gr. Cal.}}{\text{cm}^2 \times \text{sec.}} \text{ or } \frac{\text{Watts}}{\text{cm}^2}.$$

From the relation

$$\underbrace{6\,\frac{c_1}{c_2^4}\,T^4}_{\text{Second member of II}} = \underbrace{1.277 \times 10^{-12}\,T^4\,\frac{\text{Gr. Cal.}}{\text{cm}^2 \times \text{sec.}}}_{\text{Kurlbaum's Heat-equivalent}}$$

it follows that with the value $c_2 = 14455 \times$ Temperature degrees that

$$C_1 = 9292\,\frac{\text{Gr. Cal.}}{\text{cm}^2 \times \text{sec.}} \times \mu^4,$$

or

$$38710\,\frac{\text{Watts}}{\text{cm}^2} \times \mu^4.$$

With these values of the constants the equivalent in work for the radiation of the black body of Temperature T, for any region of wave-length whatever, may therefore be computed by formula I.

Minor Contributions and Notes.

THE THIRD CONFERENCE OF ASTRONOMERS AND ASTROPHYSICISTS.

The committee charged with the selection of a time and place for holding a third conference of astronomers and astrophysicists met in the city of Washington on February 8, 1899, and by unanimous vote of the members present, Messrs. Newcomb, Morley, Hale, and Comstock, resolved that such a conference should be held at the Yerkes Observatory early in the following September, the precise date to be subsequently determined by Professor Hale. In accordance with this resolution, and at Professor Hale's invitation, the conference will be held at the Yerkes Observatory, Williams Bay, Wis., beginning on Wednesday, September 6, and closing on Friday, September 8.

In its plan and scope this conference will be similar to those held in 1897 and 1898 at Williams Bay and Cambridge, accounts of which have been published in this Journal and elsewhere. The committee charged with perfecting a plan for the organization of a permanent society of astronomers and astrophysicists to have charge of future conferences will present its report at this time.

A circular giving information regarding local arrangements will be issued shortly.

Geo. C. Comstock,
Secretary.

Reviews.

Photometrische Durchmusterung des nördlichen Himmels, enthaltend alle Sterne der B. D. bis zur Grösse 7.5. Von G. Müller und P. Kempf.

Theil I. Zone 0° bis + 20° declination. 4to, pp. 501. Potsdam, 1894. Theil II. Zone + 20° bis + 40° declination. 4to, pp. 465, Potsdam, 1899.

Part I of this important work is contained in Vol. IX of the *Publicationen des Astrophysikalischen Observatoriums zu Potsdam.* The recent appearance of Part II, in Vol. XIII of the same series, calls for a review of both parts as a whole, though it is difficult to do justice, within the limits of a few pages, to such a thorough and comprehensive undertaking. The authors were at first inclined to confine their work to telescopic stars, beginning at the lower limits of the *Harvard Photometry* and *Uranometria Oxoniensis,* but finally determined upon the limits indicated in the titles to the volumes. The wisdom of this choice became more evident as the work progressed, since on its completion we will have the magnitudes of the northern naked eye stars and a large number of the telescopic ones, all measured in one homogeneous series with an accuracy not elsewhere attained. The importance of this work, both as a model and basis for future photometric studies, and as a foundation for researches on parallax and stellar evolution, can hardly be overestimated. Until the last twenty years stellar photometry has lagged far behind measures of position, and has hardly kept pace with spectroscopic studies. The authors express the opinion that in stellar astronomy measures of magnitude are of greater importance than measures of position, and that in the future, spectroscopic, colorimetric and photometric studies should go hand in hand.

The first important consideration in the plan of the work was the choice of that form of photometer best adapted to the purpose in view. Several pages of the introduction are devoted to a comparison

of the merits and demerits of a variety of instruments, and the reasons for the choice are given in so convincing a manner that no one can doubt their force. They pass over the astrometer of Herschel and the divided-objective prism photometer of Steinheil, as possessing only an historic interest; the latter, in spite of the valuable work done with it by Seidel, being out of the question on account of its limited scope, since the star images were not compared at the focus, but expanded into disks, thus confining its use to the brighter stars. The forms then left to consider are, the wedge photometer using the method of extinctions, and the two varieties of polarizing photometers using the method of equalities, the "Meridian Photometer" of Pickering, and the form devised by Zöllner. Taking these three in their order, the authors admit the many advantages possessed by the wedge photometer, holding as it does the first rank in simplicity of construction and convenience of use. It permits of measurements through a wide range of magnitudes; and, so far as depends on the determination of the point of extinction, the accuracy of the results is scarcely exceeded by any form of instrument. The great drawback to its use in an extended survey, like that proposed by the authors, is the impossibility of obtaining homogeneous glass of strictly neutral tint, all the specimens in use showing so strong a selective absorption as to cause large errors in the measurement of stars differing considerably in color. There is also the difficulty of determining accurately the constant of the wedge, and of eliminating the varying sky illumination from the measurements, so that the authors consider this form as not adapted to their purpose.

The two forms then left to consider were, first, the "Meridian Photometer," devised by Pickering, which brings the images of two stars to equality by a polarizing eyepiece; and second, the Zöllner form, in which the image of the object to be measured is compared with that of an artificial star, reduced by polarization to equality with the real one. The great advantage of the "Meridian Photometer" consists in the direct comparison of two equalized stars at the focus, a condition favorable to accuracy of measurement. Some of the disadvantages of this form are: the formation of two images by different objectives of possibly unequal transmissive power, which also may vary on account of dust, dampness, etc., the images not being as sharp as could be desired from one reflection, and the transmission of the rays through so many media; the loss of half the light by polarization;

the limitation of its use to the vicinity of the meridian; and the partial polarization caused by the reflection from the mirror used in front of the larger objectives. These many drawbacks led the authors to choose the Zöllner form in spite of the objections arising from the use of an artificial star whose image did not always closely resemble that of the real star, and the danger of variation in the intensity of the lamplight. Both these objections were carefully considered by the authors, who refer in this connection to the anomalous results obtained by Th. Wolff,[1] which in their opinion were caused by the use of too large a diaphragm, giving the artificial star a planetary rather than a stellar appearance, and causing differences of magnitude to be systematically measured too small. In the present work the authors avoided this source of error by a proper choice of diaphragms, and showed that as a result of this precaution it did not appear in their work, the proof being supplied by a series of observations in which this source of error was completely eliminated. The danger of variation in the intensity of the petroleum flame used for the artificial star was met by enclosing it so carefully as to completely protect it from currents of air, and, in addition, any possible effect was eliminated by measuring the same standard stars at the beginning, in the middle, and at the end of each zone. The authors consider that by the observance of these precautions the Zöllner photometer was best adapted to their purpose. The greater part of the work was done with two such instruments called *C* and *D*. The smaller one, *D*, was used with a Steinheil refractor of 13.5 cm aperture for stars fainter than the 6th magnitude. For the brighter stars, *C* was provided with objectives of 67 mm aperture (C_{I}), 36.5 mm (C_{II}), and 21.5 mm (C_{III}).

Of no less importance than the choice of instruments was the arrangement of the standard stars on which the catalogue magnitudes should depend, and no small part of the success of the work was due to the good judgment displayed in this particular. Polaris was considered as a standard, that the results might be directly comparable with the *Harvard Photometry* and the *Uranometria Oxoniensis*, but it was rejected for two reasons: its distance from most of the zone stars would introduce considerable and unknown errors from local irregularities in the transparency of the air; and also, since the larger part

[1] *Photometrische Beobachtungen an Fixsternen*, Leipzig, 1877; also, *Photometrische Beobachtungen an Fixsternen aus den Jahren 1876 bis 1883*, Berlin, 1884.

of the program stars were telescopic, the difference in magnitude between them and the standard would be greater than could be measured with accuracy. It was then determined to base the magnitudes on a list of 144 standard stars, distributed to the best advantage around the northern heavens. As the work planned was divided into four parts, including respectively the stars from Dec. 0° to + 20°, + 20° to + 40°, + 40° to + 60°, and + 60° to + 90°, the standards were distributed in the following manner: Nos. 1 to 48 encircled the northern hemisphere at Dec. about + 10° and at intervals of about 30 min. in R.A.; Nos. 49 to 96 in a similar way at + 30°, and the remaining 48 near + 60°. In each of these series the stars were alternately about 5.3 and 6.7 magnitude, the brighter being used with photometer *C* for the naked-eye stars, and the fainter with *D* for the telescopic stars. It was thus possible to compare each "zone" of program stars with two standards which closely included them in R. A. at about the same average Dec. and magnitude as the zone stars. This arrangement seems an ideal one for the work, making it possible to practically eliminate the systematic errors.

The thoroughness with which these standard stars were compared among themselves and a consistent system of magnitudes derived, excites our admiration. Each star was compared with its two neighbors in the same zone and with one brighter and one fainter star in each of the other zones, giving 6 values for each star and 432 combinations in all. The pairs were compared at nearly the same zenith distance and on only the best nights, so that the uncertainty in the correction for atmospheric extinction should be reduced to a minimum. The value of this correction (taken from Potsdam *Publications*, **3**, 285) to a measured difference of magnitude, never exceeded 0.13. The comparisons were mostly made with photometer *D*, with the aperture reduced so that the reading on the position circle of the Nicol prism for the fainter star was about 12°, therefore for the brighter star it did not exceed 40°, as no differences greater than 2.5 magnitudes occurred. The observations of the standards were made in series of 18 stars each, occupying 35 or 40 minutes, so that the observer was neither hurried nor wearied. The separate combinations were each measured eight times, four by Müller and four by Kempf. If the extremes of the values thus found differed by as much as 0.4 mag., the pair was remeasured by one observer, if by more than 0.5 mag., by both. When it is remembered that such a difference included the effect of

instrumental errors, of personal equation between the observers, and most dangerous of all, the effect of local inequalities in the atmospheric extinction, since the stars were often observed at widely varying azimuths, the precision of the results furnishes convincing evidence that these errors were well eliminated. On account of these precautions revision was necessary in only 93 cases out of 432. A difference greater than 0.3 mag. between a single measure and the mean of 8 occurred but 11 times in 3564 observations, less than ⅓ of 1 per cent., and *there was no residual greater than 0.4 mag.* The one source of possible error in this arrangement would seem to lie in the fact, already mentioned, that the corrections for extinction were taken from a mean table prepared for Potsdam, instead of being deduced from separate observations for each night. In the *Annals of Harvard College Observatory*, **23**, 91, Professor Pickering gives the separate values for the coefficient of extinction for different nights at Cambridge. The mean value is 0.25, but it varies from 0.19 to 0.90, though the statement is made that when the coefficient is above 0.50 the air is distinctly hazy. Such a large possible variation would seem to render the use of a mean value of doubtful propriety, or at least restrict the observations to the very best nights. That this latter condition is strictly fulfilled in the work under review is completely proven by the agreement of the separate values, the probable error of a single measured difference of magnitude being ± 0.073, that of the mean of 8 only ± 0.027, a degree of accuracy which attests the high rank to which the work is entitled.

The problem of deducing from these differences a consistent system of magnitude for the standard stars was solved by a series of approximations. The *B. D.* magnitudes were substituted in the six equations for each star and the means were again used as a second approximation. This process was continued till a further substitution would make no change in the result. Thus the two systems were made to exactly correspond, the average magnitude of the 144 stars being 6.02, both with the Potsdam and *B. D.* values. The individual differences varied from — 0.76 mag. to + 0.89 mag., which was considered by the authors to be due almost entirely to accidental errors in the *B. D.* estimates. The differences between the two systems, taken without regard to sign, averaged 0.27, and the sum of all the differences was 0.00. The authors are therefore led to believe that the relative brightness of the fundamental stars is determined within 0.05 mag.

An instructive result from the observations of the standard stars was the discovery that one of them, No. 56 (*B. D.* + 30° 591), was a variable of unusually slow rate of change. It was observed as a fundamental star 48 times, between 1888 February 25 and 1891 February 13, with very consistent results, the mean being 6.31 mag. It was used as a comparison star with the zones in Part II from 1890 November 22 to 1895 November 16, when its variability was first recognized, and then followed as a variable till 1898 March 20. The results are given in detail in *Astronomische Nachrichten*, **146**, 170, 1898,[1] and the following table of means is taken from Part II, p. 7:

Interval of time							Number of observations	Mag.
1888	Feb.	25	to	1889	Feb.	28	20	6.30
1889	Nov.	9	"	1890	Mar.	13	14	6.33
1890	Oct.	1	"	1891	Feb.	13	16	6.30
1893	Nov.	9	"	1894	Mar.	24	5	6.44
1895	Feb.	8	"	1895	Apr.	2	9	6.60
1895	Sept.	26	"	1896	Apr.	2	41	6.69
1896	Aug.	28	"	1897	Apr.	9	28	6.82
1897	Aug.	18	"	1898	Mar.	20	26	6.92

The zones with which this star was used were reduced with the other standard, No. 54 or 58.

Section 1 of Part I contains the separate results of the measures of each of the 432 pairs of standard stars, besides several tables giving method of reduction and the agreement of the resulting magnitudes.

Section 2 of Part I gives the observations in detail for the Zone 0° to + 20°. Every observed quantity is given, and nearly all those used in the reductions, so that the entire work can be easily checked. The arrangement is so excellent, and will be so useful as a model for future work, that it seems worth while to describe it fully. The amendments to the plan which were introduced in Section 1 of Part II will be mentioned in their proper place.

In each part of the work the program stars were divided into three groups, according to their brightness, from 2 to 4, from 4 to 6, and from 6 to 7.5 magnitudes, respectively. The first two groups were observed with photometer *C* and the fainter group with *D*. For convenience in use of the equatorially mounted refractor carrying photometer *D*, the third group was again divided into two sub-groups, according to declination, from 0° to + 10° and + 10° to + 20° in Part I,

[1] See also *A. J.*, **20**, 45.

and from $+20°$ to $+30°$, and $+30°$ to $+40°$ in Part II. Within these groups the stars were divided into zones of twelve each in Part I, which number was increased to fourteen in Part II. For each of these zones two standard stars were selected, which included it in R. A. and did not differ greatly in declination from the mean of the zone stars. The odd numbered standards, averaging 5.3 mag., were used in the measures with photometer *C*, and those with even numbers, averaging 6.7 mag., with *D*. By following this carefully considered plan, it was possible to observe the stars of a given zone at nearly the same zenith distance, generally between 40° and 50°, so that the correction to a measured difference in magnitude on account of the difference in zenith distance was usually less than 0.1 mag., and never greater than 0.2 mag.; and since the azimuths were so nearly alike the uncertainty in this correction from local irregularities in absorption were quite negligible. Thus the greatest element of uncertainty in photometric work was eliminated. Of somewhat less importance was the danger, inherent in the Zöllner photometer, of under-measuring differences in magnitude when the interval is large. This was avoided by the method of grouping the zone stars and using with each group the standards of similar brightness. In this way intervals greater than 2.5 mag. were nearly always avoided, in fact they were usually less than 1.0 mag. It is evident that the eminent success of the work is due in a great measure to the proper choice of the zone and standard stars, both as regards position and magnitude. The observing program, which was strictly adhered to, was as follows: Two standard stars, six zone stars, the same standards again, six more zone stars, and finally the two standard again. Müller and Kempf alternated as observer and recorder, and by this coöperation the time occupied on a zone was usually thirty-five minutes, giving ample time for certain identification and deliberate settings of the photometer circle in each of the four quadrants. As each series was short, the observer's eye was not wearied, and his perception of the equality of the images was not dulled at the end of the zone.

It was the original intention to observe each star four times, but it was soon found that a sufficient accuracy would be obtained by two. When these results differed by less than 0.3 mag. the mean was taken as the catalogue value. The discrepancy exceeded this limit in the case of 126 stars in Part I, and 161 in Part II, being 3.6 per cent. in each. These stars were arranged in "Revision Zones," and re-measured by both observers. The colors of the stars were taken from the *Potsdam*

Spectroscopic Durchmusterung for Part I, but for Part II independent estimates were made, dividing the interval between white and red into 18 steps. The allowable margin of divergence was placed at 4 steps, which was exceeded and revision required in case of 107 stars, or 2.4 per cent. of the whole number.

The observations for Part I were made on 405 nights between 1886 October 1, and 1893 April 1, for Part II on 276 nights between 1890 September 18 and 1898 May 2. The number of program and revision zones was 601 and 59 in Part I, and 685 and 54 in Part II, resulting in catalogues of 3522 and 4416 stars, respectively. The heading of each zone gives the current number, date, photometer used, observer, list numbers of the two standard stars, and the quality of the seeing, expressed with the notation 1–4, the condition 1 meaning "unusually clear and quiet," and 4 meaning "medium." (In Part II these are explained as "excellent," and "bad," respectively.) The seeing was marked 4 in case of 5 zones in Part I, and 2 zones in Part II. In each zone the first column gives the *B. D.* number of the star, then follows the sidereal time of the observation, the four settings of the photometer circle to 0.1°, then the quantity J, which is the mean of the settings, the zenith distance to 0.1°, the quantity $\log \sin^2 J$, and the same corrected for atmospheric extinction by the table in Potsdam III, 285. The extreme simplicity of the reductions arises from the light of the stars being proportional to the quantities $\sin^2 J$, and accepting, as the authors do, Pogson's ratio of magnitudes, the number whose log is 0.4, the differences in brightness are converted into magnitudes by dividing the differences in $\log \sin^2 J$ by 0.4. At the foot of each zone are given the three separate means of the $\log \sin^2 J$ for the standard stars observed at the beginning, middle and end, also the general mean of the three which is used in the reductions. Column 11 gives the difference between this logarithm and those for the zone stars; column 12, the difference reduced to magnitudes by dividing by 0.4, and the last column, the resulting magnitudes of the zone stars, obtained by applying these differences to the mean of the values for the two standards. Footnotes relate to the measurements of double stars, and any unusual condition of the atmosphere.

A good idea of the consistency and accuracy of the results can be gathered from a comparison of the three sets of measurements of the fundamental stars with each zone. Any change in the intensity of the lamp flame, or dulling of the observer's eye, would appear in a regular

progression in the residuals obtained by subtracting the general from the three separate means. This was done in Part I with the following results:

	Number of residuals	
	Positive	Negative
Beginning of the zone - - -	319	303
Middle of the zone - - -	292	330
End of the zone - - - -	323	299

The algebraic sum and average value of the residuals were as follows:

	Sum of residuals	Average value
Beginning - - - -	+2.83	+0.005
Middle - - - -	−3.77	−0.006
End - - - - -	+1.05	+0.002

It appears from these figures that no progressive errors of the kind indicated were present in the work. The authors do not give the absolute values of these residuals, but a partial idea of their magnitude was gained from 36 zones, equally distributed over Parts I and II. The mean value was 0.031 mag. and 0.043 mag., and the largest individual differences 0.089 mag. and 0.138 mag. for the two parts respectively, the degree of accuracy shown being in good agreement with the results given by the authors. The probable error of the measurement of a pair of standard stars was ± 0.046 for photometer *C* and ± 0.038 for *D;* that of a single standard star was ± 0.065 and ± 0.053 for the same instruments. A comparison was also made of the magnitude differences of the pairs of standard stars from the zone work, with the values obtained from the original comparisons of these stars among themselves. The results are given in detail for each of the 96 stars in Parts I and II, and separately for the two photometers. The mean difference found by subtracting the latter from the former, for *C* and *D* in their order, was + 0.010 and — 0.010 in Part I and — 0.003 and + 0.005 in Part II. The absolute values without regard to sign averaged 0.056 and 0.059 in the two parts. Finally the difference between the observers in the sense *M*—*K* was + 0.022 for *C* and + 0.035 for *D* in Part I, and — 0.02 for each in Part II. It seems to the reviewer that a consideration of these extremely consistent results will do much to give Stellar Photometry a place among the exact sciences, to which, until lately, it could lay no claim.

The Catalogues themselves contain 50 stars on a page. The first five columns are alike in Parts I and II, giving the current number, the *B.*

D. number, the R. A. and Dec. reduced to 1900, and the *B. D.* magnitude. In Part I there are six more columns, containing the separate magnitude results of Müller and Kempf, the numbers of the zones from which they were taken, unless the star was observed more than twice, the color, the adopted magnitude, being the mean of *M* and *K*, and finally Bayer's letter or Flamsteed's number of the star, or for doubles, Struve's number. In place of these six columns there are four columns in Part II, giving respectively the zones in which the star occurs unless it was observed more than four times, the color, the adopted mean magnitude, and the designation from Bayer, Flamsteed or Struve.

The "Conclusions," filling about twenty pages at the end of each part, contain several very important discussions which, however, do not readily lend themselves to condensation for the purpose of review. Only brief mention can be made of their contents. The first is a comparison of the different photometers, leading to very interesting but not entirely satisfactory conclusions. In Part I there were 240 stars measured both with *C* (aperture 67 mm) and *D* (aperture 135 mm). In Part II, 143 stars were thus compared. This was done for the purpose of testing both the homogeneousness of the work and the presence of the tendency, peculiar to the Zöllner photometer, towards measuring differences of magnitude too small. The result in Part I was that the difference between the two instruments was vanishingly small and exhibited no progression. In Part II the difference *D*–*C* decreased from + 0.12 for stars of magnitude 5.27 to — 0.01 for magnitude 6.11. The divergence was in the direction to be expected. A similar test of 51 stars with photometers C_{II} and C_{III} (apertures 36.5 and 21.5 mm) showed differences varying from — 0.04 to + 0.03, the progression being in a direction opposite to that in the former test. The authors conclude that the peculiar tendency of this form of photometer manifests itself only with the larger apertures in which the images of the brighter stars are diffuse.

The comparison between observers was extremely satisfactory. In Part I the difference *M*–*K* varied from — 0.14 for stars brighter than 3.0 to + 0.11 for those fainter than 8.0, and excluding the extremes of magnitude the divergence was only from — 0.05 to + 0.06. In Part II the agreement was much closer, the differences lay between — 0.03 and + 0.03, the slight progression manifest in Part I entirely disappearing. The absolute values of the difference in both parts

was 0.12, giving for the probable error of a single observation ± 0.057, that of a catalogue magnitude from the mean of two measures, ± 0.040, which speaks volumes for the adaptability of the instrument and the skill of the observers. The above comparison is carried out separately for stars of the different colors, the results showing the personal equation between the two to be very small.

A comparison of greatest importance is given between the Potsdam and Bonn *Durchmusterungs*. The principal results appear in the following table, which is extracted from Table VI, page 454, Part II.

Magnitudes	Part I		Part II	
	Number of stars	*P.-B. D.*	Number of stars	*P.-B. D.*
< 3.0	9	+ 0.55 m	12	+ 0.34 m
3.0 to 3.4	21	+ 0.42	24	+ 0.22
3.5 3.9	26	+ 0.36	15	+ 0.22
4.0 4.4	42	+ 0.19	46	+ 0.25
4.5 4.9	45	+ 0.08	64	+ 0.13
5.0 5.4	108	+ 0.04	126	+ 0.05
5.5 5.9	167	— 0.06	164	— 0.01
6.0 6.4	327	+ 0.01	391	+ 0.09
6.5 6.9	658	+ 0.06	912	+ 0.19
7.0 7.4	1252	+ 0.02	1628	+ 0.26
7.5	680	— 0.03	904	+ 0.27

For the naked-eye stars there is a fair agreement between Parts I and II, showing that a difference of 1.0 mag., *B. D.* is equivalent to 0.823 mag. and 0.905 mag. in the Potsdam system and giving as the log of the *B. D.* magnitude ratio 0.329 and 0.362 in the two parts respectively. For the telescopic stars the divergence is much greater, the resulting logs of the ratio being 0.400 and 0.457. Fortified by other photometric researches the authors consider these differences to be due to a lack of homogeneousness in the *B. D.* estimates.

The final comparison is that between the Potsdam results and those of the three other photometric catalogues, the *Harvard Photometry* (*H. C. O. Annals*, Vol. XIV), called here "Pickering I," the Photometric Revision of the *Durchmusterung* (*H. C. O. Annals*, Vol. XXIV), called "Pickering II," and the *Uranometria Nova Oxoniensis*, called "Pritchard." Part I contains a close criticism of the three catalogues, and Part II gives the results of a comparison of the systems, arranged according to the color and magnitudes of the stars. The mean results

are given here, condensed from Table VIII, page 459, the differences being taken in the sense Potsdam — the others :

	Part I		Part II	
	Number of stars	Diff.	Number of stars	Diff.
Pickering I	791	+ 0.17 m	799	+ 0.18 m
Pickering II	801	+ 0.13	984	+ 0.16
Pritchard	691	+ 0.13	686	+ 0.14

No progression with the magnitude is shown except in the comparison with Pickering II, where it is indicated that 1.0 mag. in the Potsdam system is equivalent to 1.062 mag. in the system of the "Photometric Revision." But when arranged according to color the case is entirely different, so much that if two stars, one white and the other yellow, were measured equal at Cambridge or Oxford, the yellow star would appear 0.3 mag. or 0.4 mag. brighter than the white in the Potsdam measures. While accounting for part of the difference by the "Purkinje" phenomenon, the authors do not consider the question as finally settled.

The appearance of the remaining parts of the work will be awaited with great interest, and one cannot but echo the hope expressed by the late lamented Lindemann, that when completed the catalogue may be issued separate from the zone observations.

J. A. Parkhurst.

Kollektivmaasslehre von Gustav Theodor Fechner, im Auftrage der königl. Sächsischen Gesellschaft der Wissenschaften herausgegeben von Gottl. Friedr. Lipps. Leipzig, 1897. (VIII + 483 pages, 8vo.)

Gauss's well-known formulae for the discussion of errors of observation constitute the so-called method of least squares. The chief use of this method is to furnish some criterion for the divergencies of observations that ought not to be regarded as due to ordinary accident. These when sufficiently numerous lead to careful search for their proximate cause, and in some cases to the discovery of previously unknown causes or laws. Gauss's reasoning is not, however, as many seem to think, deductive and necessarily valid ; it rests on two assumptions ; first, that the arithmetic mean of empiric data is the most probable value ; second, that a positive error and a negative error of equal

numerical magnitude are equally probable. These assumptions Fechner calls in question, and attempts a recasting of the theory.

Not alone observations of physical or astronomical magnitudes, but statistics of all sorts are to be treated. For simplicity it is presumed that the data refer to things nearly enough alike to be classified together. For such sets of objects the name *Kollektiv-Gegenstände* is introduced. If the Gaussian hypothesis were applicable, the measurements of a sufficiently numerous set ought to lie half above, half below the arithmetical mean, *i. e.* the distribution would be symmetrical, and increasing the number ought to bring about a closer approximation to symmetry. Now in some sets of data Fechner found such symmetry lacking, and the asymmetry grew more evident with increasing number of data. For such cases, and he believes them to be more common than the symmetrical kind, Fechner proposes to regard as the most probable value *a priori* that which may be called the *densest* value, that value which corresponds to a maximum ordinate in the curve of frequency. Variations are commonly assumed to be small as compared with the magnitudes measured; but for the case of large variations his proposal is to use the logarithms instead of the natural numbers. On this basis is derived his "Generalized Gaussian Law," applicable to all sets of data where the variations are properly considered fortuitous, even those essentially asymmetrical and showing wide variation, if the curve of frequency presents only a single maximum.

Among the consequences of the generalized law perhaps the most interesting is the relation between number and magnitude of positive and negative variations from the density center. In the symmetrical case, the number of positive variations and their sum would exactly equal those of negative variations; in the general case this equality is replaced by proportionality, and the number of variations must be squared:

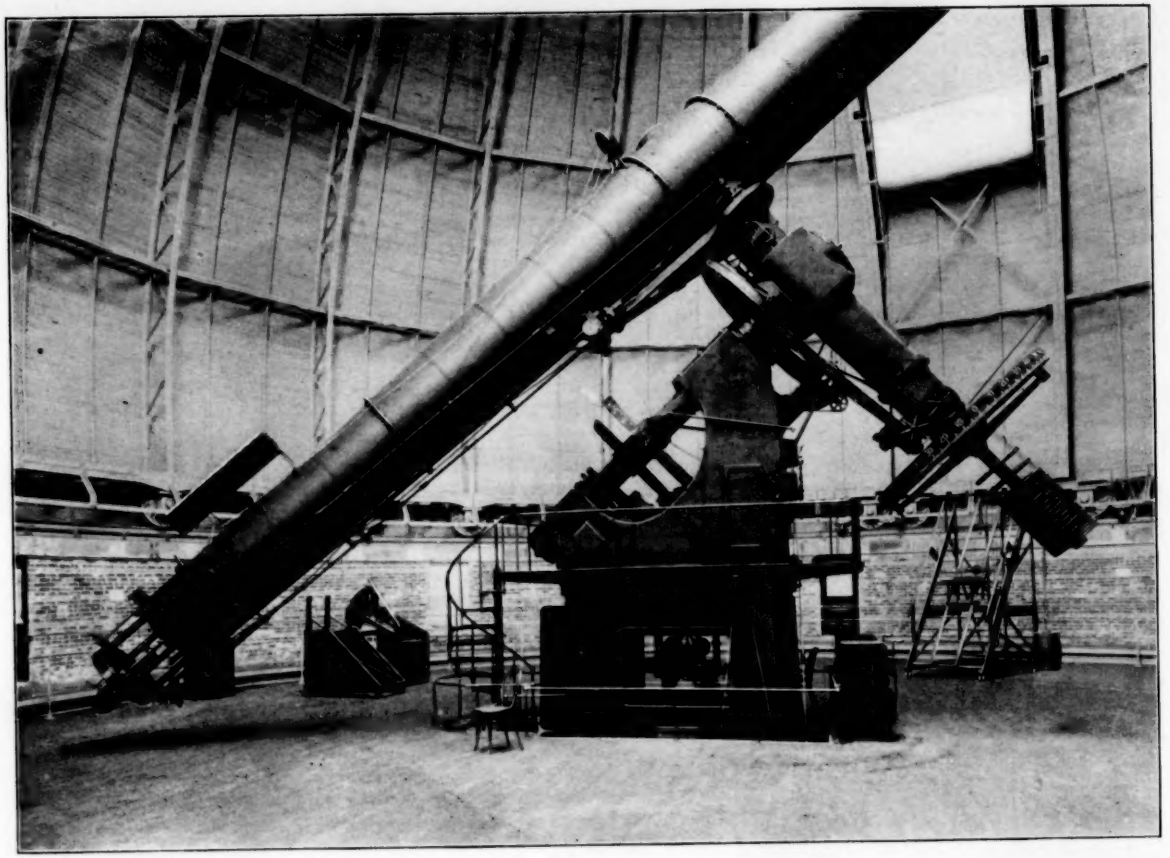
$$m'^2 : m_1^2 :: \Sigma\delta' : \Sigma\delta_1.$$

This property of the density-center may be adopted as the mathematical definition of the term, as Fechner remarks (p. 70). Otherwise the center of density must be found either roughly by platting or more exactly by the use of a series to represent the frequency-function, whose maximum will then be determined by the usual method. Explicit rules are given for all the operations described, and practical illustrations fully worked out.

As indicating the broad empirical basis of this work we note the range of statistical material employed, (pp. 28–29): Saxon recruits (over 16000), skull measurements (500), weights of internal organs of the human body, 350 heads of rye, records of daily temperature and rainfall, dimensions of *cartes de visite*, and of frames in several galleries of paintings. Among many valuable additions by the editor is a discussion based on data from the Strassburg Observatory, showing that errors of observation do not require the hypothesis of essential asymmetry (p. 456). The fact that most summer days are cooler than their average, winter days warmer, confirms the general belief. Interesting facts and suggestions are sprinkled liberally through the book, which is written, moreover, in a style as free as possible from mathematical technicalities. For one wishing to form a hasty acquaintance with the work, the first six chapters (98 pages) are especially to be recommended.

HENRY S. WHITE.

PLATE I.



THE FORTY-INCH TELESCOPE AND STELLAR SPECTROGRAPH.